The 1973 Least-Squares Adjustment of the Fundamental Constants*

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This paper is a summary of the 1973 least-squares adjustment of the fundamental physical constants carried out by the authors under the auspices of the CODATA Task Group on Fundamental Constants. The salient features of both the input data used and its detailed analysis by least-squares are given. Also included is the resulting set of best values of the constants which is to be recommended for international adoption by CODATA, a comparison of several of these values with those resulting from recent past adjustments, and a discussion of current problem areas in the fundamental constants field requiring additional research.

Key words: Data analysis; fundamental constants; least-squares adjustments; quantum electrodynamics.

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Glossary of Symbols and Units[1]

a_e	Magnetic moment anomaly of the free electron: $a_e = (g_e - 2)/2$
a_p	Magnetic moment anomaly of the free proton: $a_p = \mu_p/\mu_N - 1$
a_μ	Magnetic moment anomaly of the free muon: $a_\mu = (g_\mu - 2)/2$
A	Absolute ampere: The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
Å	ångström (10^{-10} m)
Å*	ångström-star x-ray unit defined by $\lambda(WK\alpha_1) \equiv 0.2090100$ Å*
A_{B169}	BIPM realization of the ampere on 1 January 1969: $A_{B169} = V_{B169}/\Omega_{B169}$
BIPM	Bureau International des Poids et Mesures
c	Speed of light in vacuum
g	Acceleration due to gravity
e	Elementary charge
ETL	Electrotechnical Laboratory, Japan
F	Faraday constant: $N_A e$
g_e	g-factor of the free electron: $g_e = 2\mu_e/\mu_B$
g_p	g-factor of the free proton (referred to the Bohr magneton): $g_p = 2\mu_p/\mu_B$
g_j (H)	g-factor of the electron in the ground state of hydrogen
g_p (H)	g-factor of the proton in the ground state of hydrogen
g_p (H_2O)	g-factor of protons in H_2O (spherical sample)
g_μ	g-factor of the free muon: $g_\mu = 2\mu_\mu/(e\hbar/2m_\mu)$
g'_μ	g-factor of the free muon (referred to the Bohr magneton): $g_\mu = 2\mu_\mu/\mu_B$
G	Newtonian gravitational constant
h	Planck constant
Hz	hertz (cycle per second)
IGSN71	1971 International Gravity Standardization Net
IMM	Mendeleev Institute of Metrology, U.S.S.R. (see VNIIM)
k	Boltzmann constant: R/N_A
kg	kilogram: The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
kxu	kx-unit based on $\lambda(CuK\alpha_1) \equiv 1.537400$ kxu
K	kelvin: The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
K	The ratio A_{B169}/A
KhGNIIM	Kharkov State Scientific Research Institute of Metrology, U.S.S.R.
m	metre: The metre is the length equal to 1650763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom.
mol	mole: The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.
m_e	Electron rest mass
M_e	Atomic mass of the electron (relative to ^{12}C: $M_e = m_e/m_u$)
m_p	Proton rest mass
M_p	Atomic mass of the proton (relative to ^{12}C: $M_p = m_p/m_u$)
m_u	Unified atomic mass constant: $m_u = m(^{12}C)/12 = 1$u
m_μ	Muon rest mass
mGal	10^{-5} m·s^{-2}
N_A	Avogadro constant
NBS	National Bureau of Standards, U.S.
NPL	National Physical Laboratory, U.K.
NSL	National Standards Laboratory, Australia
ppm	parts per million
PTB	Physikalisch-Technische Bundesanstalt, Germany
r_i	Residual of a particular input datum in a least-squares adjustment
R_B	Birge ratio
R	Molar gas constant: p_0V_m/T_0
R_∞	Rydberg constant for infinite mass: $\alpha^2/2\lambda_C$
RSS	Square root of the sum of the squares, or root-sum-square

[1] Base Unit definitions were taken from ref. [25.1].

Note: The subscripts in the symbols μ_B, μ_N, m_e, etc. are italicized in this paper in accordance with American Institute of Physics practice. However, it should be pointed out that the recommendations of ISO, ANSI, and several other organizations call for certain of these subscripts to appear in roman type.

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s	second: The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
SI	Système International d'Unités, the official name of the system of units based on the metre, second, kilogram, ampere, kelvin, candela, and mole.
$\mathcal{S}$	Lamb shift in hydrogenic atoms ($nP_{1/2} - nS_{1/2}$ interval)
T	tesla (one tesla $\hat{=}$ 10^4 gauss)
T_0	0°C on the thermodynamic temperature scale: $T_0 = 273.15$ K
u	Atomic mass unit [unified scale, $1\text{u} = m(^{12}\text{C})/12 = 10^{-3}\ \text{kg}\cdot\text{mol}^{-1}N_A^{-1}$]
V_m	Molar volume of an ideal gas at s.t.p.
V_{B169}	Defined 1 January 1969 BIPM as-maintained volt: $2e/h \equiv 483594.000\ \text{GHz}/V_{B169}$
VNIIM	All-Union Scientific Research Institute of Metrology (Mendeleev Institute, U.S.S.R.)
WQED	Without quantum electrodynamic theory
α	Fine structure constant: $[\mu_0 c^2/4\pi](e^2/\hbar c)$
α^{-1}	Inverse fine structure constant
γ_p	Gyromagnetic ratio of the free proton
γ'_p	Gyromagnetic ratio of protons in H_2O (spherical sample)
γ'_p(low)	γ'_p obtained by the weak or low field method
γ'_p(high)	γ'_p obtained by the high field method
λ_C	Compton wavelength of the electron: $h/m_e c$
ΔE	Fine-structure splitting in hydrogenic atoms ($nP_{1/2} - nP_{3/2}$ interval)
Λ	Ratio, kx-unit to ångström
Λ^*	Ratio, Å* to ångström
μ_e	Magnetic moment of the free electron
μ_B	Bohr magneton: $e\hbar/2m_e$
μ_N	Nuclear magneton: $e\hbar/2m_p$
μ_p	Magnetic moment of the free proton
μ'_p	Magnetic moment of protons in H_2O (spherical sample)
μ_μ	Magnetic moment of the free muon
μ_0	Permeability of vacuum ($4\pi \times 10^{-7}\ \text{H}\cdot\text{m}^{-1}$)
ν	Degrees of freedom in a least-squares adjustment
ν_{01}	$2\,^3P_0 - 2\,^3P_1$ fine structure interval in atomic helium
ν_{Hhfs}	Ground-state hyperfine splitting in hydrogen
ν_{Mhfs}	Ground-state hyperfine splitting in muonium
ν_{Pshfs}	Ground-state hyperfine splitting in positronium
σ	Stefan-Boltzmann constant: $\pi^2 k^4/60\hbar^3 c^2$
$\sigma(H_2O)$	Diamagnetic shielding correction for protons in H_2O (spherical sample)
χ^2	The statistic "chi squared"
Ω_{B169}	BIPM realization of the ohm on 1 January 1969.

I. Introduction

The extraordinary amount of new experimental and theoretical work which has been completed since the appearance of the comprehensive review and least-squares adjustment of the fundamental physical constants by Taylor, Parker, and Langenberg [0.1][2] in 1969 necessitates a new review and recommended set of best values. Under the auspices of the CODATA Task Group on Fundamental Constants,[3] we have completed such a review and least-squares adjustment.[4] Here, we summarize the more important aspects of the input data used and its analysis, as well as give the resulting set of best values of the constants which is to be recommended by CODATA for official international adoption and use. For completeness, we also include a set of constants derived from input data that do not require quantum electrodynamic theory for their analysis.

II. Review of Data

In this, the major portion of the paper, we review all of the data currently available that relate in one way or another to a least-squares adjustment of the fundamental constants. The review is divided into three parts:

A. The More Precise Data
B. The Less Precise WQED Data
C. The Less Precise QED Data

Here, as in ref. [0.1], WQED stands for "without quantum electrodynamic theory." However, it should be noted that all of the data to be discussed in A fall into this category as well, that is, it is not essential to use quantum electrodynamic theory for their analysis. The exact meaning of the terms "More Precise" and "Less Precise", as well as the motivation for following Taylor et al.'s practice of dividing the data into two parts, WQED and QED, will be given in the introductory remarks in portions A, B, and C.

A. The More Precise Data

In general, the input data used in a least-squares adjustment of the constants are classified into two groups. The first group, known as the auxiliary

[2] Figures in brackets indicate literature references at the end of this paper.

[3] CODATA (Committee on Data for Science and Technology) is under the jurisdiction of the International Council of Scientific Unions (ICSU). The membership of the CODATA Task Group on Fundamental Constants is: E. R. Cohen (Chairman); R. D. Deslattes; H. E. Duckworth; A. Horsfield; B. A. Mamyrin; B. N. Oleinik; H. Preston-Thomas; U. Stille; J. Terrien; and Y. Yamamoto.

[4] A progress report was presented by the authors at the Fourth International Conference on Atomic Masses and Fundamental Constants, Teddington, England, 1971, and appears in the Conference Proceedings (ref. [0.2]).

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constants, contains quantities which have uncertainties sufficiently small that they can be considered as exactly known. The second group contains the more imprecise or stochastic input data. The latter are the quantities subject to adjustment and from which are chosen the several unknowns or "adjustable constants" in terms of which the least-squares calculations are actually carried out. In the past, a quantity with an uncertainty of several tenths of a part-per-million (ppm) could be safely used as an auxiliary constant, while most stochastic data had uncertainties of at least several ppm. However, the increased accuracy with which many of the fundamental constants may now be determined has narrowed the distinction between auxiliary constants and stochastic input data. Thus, for the present discussion we choose to divide the data into two categories, "The More Precise Data" and "The Less Precise Data", with the dividing line at about the 0.5 part-per-million (ppm) level. Nevertheless, the term "auxiliary constant" will still be used to mean a quantity which may be assumed to be exactly known, that is, one with an uncertainty which is negligible compared with the uncertainties of other quantities that might appear with it in the same equations. (By negligible is meant at least a factor of three less, and in most cases a factor of between five and ten less, than the uncertainties associated with these other quantities.) Similarly, the term "stochastic data" will still be used to refer to those quantities that are subject to adjustment, that is, ones for which the input and output values will generally differ.

It should be noted that all uncertainties quoted in this paper are meant to correspond to one standard deviation, and that the notation, our handling of numerical results, and other general aspects of the analysis are more or less the same as in Taylor et al. [0.1].

1. 2e/h From the ac Josephson Effect

Several of the national laboratories are now routinely carrying out measurements of $2e/h$ by the ac Josephson effect with an accuracy of a few parts in 10^7 or better. Indeed, the U.S. National Bureau of Standards (NBS), on 1 July 1972, adopted the exact value $2e/h$ = 483593.420 GHz/V_{NBS} for use in maintaining the U.S. legal or as-maintained volt [1.1, 1.2]. Table 1.1, which is taken in part from the review paper of Eicke and Taylor [1.3], summarizes the present situation. (We have also included the 1970 measurements of Finnegan et al. [1.4] at the University of Pennsylvania; of Petley

TABLE 1.1. Summary of $2e/h$ measurements via the ac Josephson effect.[a] (The 1971 and 1972 values were obtained during the period of a series of direct volt transfers between the National Bureau of Standards and the participating laboratories.)

Lab[b]	$2e/h$ (GHz/V_{LAB})	Uncertainty (ppm)	Approximate time period of measurements	Assumed exact mean time of measurements
		1970		
U. Pa.[c]				
(NBS Units)	483593.718(60)	0.12	Feb.—May	1970.33
NPL[d]	483594.2(4)	0.8	Jun.	1970.50
NSL[e]	483593.84(5)	0.1	Jun.—Jul.	1970.52
PTB[f]	483593.7(2)	0.4	Fall	1970.79
		1971		
NBS[g]	483593.589(24)	0.05	Jul.—Aug.	1971.57
NPL[h]	483594.15(10)	0.2	Jul.	1971.58
NSL[i]	483593.80(5)	0.1	Jun.—Jul.	1971.49
		1972		
NBS[j]	483593.444(24)	0.05	Apr.	1972.29
NPL[k]	483594.00(10)	0.2	Apr.	1972.28
NSL[l]	483593.733(48)	0.1	Mar.—Apr.	1972.26
PTB[m]	483593.606(19)	0.04	May	1972.38

[a] Ref. [1.3]. [b] NBS = National Bureau of Standards, U.S.; NPL = National Physical Laboratory, U.K.; NSL = National Standards Laboratory, Australia; PTB = Physikalisch—Technische Bundesanstalt, Germany. [c] Ref. [1.4]. [d] Ref. [1.5]. [e] Ref. [1.6]. [f] Ref. [1.7]. [g] Ref. [1.8]. [h] Ref. [1.9]. [i] Ref. [1.10]. [j] Ref. [1.11]. [k] Ref. [1.12]. [l] Ref. [1.13]. [m] Ref. [1.14].

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TABLE 2.1. Results of the 1970 Bureau International des Poids et Mesures (BIPM) international intercomparison of the units of emf and resistance as maintained by various countries and BIPM (central date: 1 February 1970); and the changes made by the various countries in their as-maintained units of emf and resistance on 1 January 1969.[a] [$X_{LAB} = X_{BIPM} + \Delta\mu X$, and X_{LAB} (Post 1/1/69) = X_{LAB} (Pre 1/1/69) $+\Delta\mu X$, where X = V or Ω.]

Lab	Country	1970 BIPM comparison		1 January 1969 changes	
		$\Delta\mu V$	$\Delta\mu\Omega$	$\Delta\mu V$	$\Delta\mu\Omega$
DAMW	E. Germany	2.49	0.10	0	0
PTB[b]	W. Germany	−0.26	0.33	−10.4	−5.1
NBS	U.S.A.	0.17	0.03	−8.4	0
NSL	Australia	0.00	0.29	−16.2	3.8
NRC	Canada	0.10	−0.47	−8.0	2.7
LCIE	France	0.23	0.30	−6.1	12.2
IEN	Italy	0.04	0.78	−10.1	0
ETL	Japan	0.51	−0.19	−8.3	0
NPL	Great Britain	0.69	0.31	−13.0	3.7
IMM[c]	U.S.S.R.	2.16	−0.01	−16.0	0
BIPM		0	0	−11.0	0

[a] Ref. [2.1].
[b] A direct transfer between PTB and BIPM using a shippable, temperature regulated enclosure was carried out during the period July to September, 1972 (23 August central date) with the result $V_{PTB} - V_{BIPM} = 0.31\ \mu V$ [2.2].
[c] The U.S.S.R. volt was actually changed on 1 January 1970; see ref. [2.3].

and Gallop [1.5] at NPL; and of Harvey et al. [1.6] at NSL.) In the table, and throughout the paper, V_{LAB} means the unit of voltage of the laboratory in question as maintained at the time of the measurement. The quoted uncertainties contain both random and systematic components and are as given by the experimenters. The 1971 and 1972 results were obtained during a series of direct transfers between NBS and the participating laboratories, the purpose of which was to provide a sound basis for comparing values of $2e/h$. These transfers were carried out under the auspices of the Bureau International des Poids et Mesures (BIPM), and utilized shippable temperature-regulated volt transport standards. The volt differences as obtained from these direct transfers will be given in the next section. The last column of table 1.1 gives the assumed exact mean times of the various $2e/h$ measurements. They will be used in a least-squares analysis (also to be described in the next section) of the time dependence of the as-maintained units of voltage of the participating laboratories as implied by the $2e/h$ and volt comparison data.

2. Differences in As-Maintained Units of Voltage and a Value of $2e/h$ in BIPM Units

The results and central dates of the triennial BIPM intercomparisons of the as-maintained units of voltage and resistance of the various national laboratories for the period 1950 through 1967 are summarized in tables I, II, and III of ref. [0.1]; they will not be repeated here. However, we do give in table 2.1 the results of the 1970 BIPM intercomparisons (central date: 1 February 1970); and the 1 January 1969 changes made by the various countries in their as-maintained units of voltage (and resistance) in order to bring them into better agreement with the Système International d'Unités (SI) absolute volt (and ohm) [2.1]. In table 2.2 we give the results of the direct transfers carried out by NBS under the auspices of BIPM during 1971 and 1972 [1.3]; and for comparison purposes the volt differences implied by the $2e/h$ measurements given in table 1.1. We have somewhat arbitrarily assigned the same uncertainty to the triennial intercomparison results, that is, 0.14 μV, as has been assigned the good direct transfers carried out with temperature-regulated volt transport standards. Overall, the data are reasonably consistent.[5]

On the basis of the $2e/h$ data of table 1.1 and an analysis carried out by Denton of NPL, the Comité Consultatif d'Electricité (CCE) of the Comité International des Poids et Mesures (CIPM), at its 13th meeting (held October 1972), adopted a resolution (Statement E–72) which was subsequently approved by the CIPM at its 61st meeting (also held October 1972), which reads in part [2.4]:

"*Considers* from these results that, on 1st January 1969, V_{69-BI} was equal within half a part per million to the potential step which would be produced by a Josephson junction irradiated at a frequency of 483594.0 GHz."

In view of the CCE statement, the increased number of precision measurements involving electrical units which have been carried out in national laborato-

[5] With the uncertainties assigned in table 2.2, and even though they are probably too small rather than too large, the apparent 0.6 μV discrepancy in the 1970 BIPM intercomparison result for $V_{BIPM} - V_{NBS}$ pointed out in ref. [1.3] tends to disappear.

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TABLE 2.2. Summary of differences in units of voltage maintained by several national laboratories and NBS, $V_{LAB} - V_{NBS}$[a,b]

LAB	1970 Triennial international comparison (μV_{BIPM})	1970		1971		1972	
		Direct volt transfers (μV_{NBS})	From values of $2e/h$ (μV_{NBS})	Direct volt transfers (μV_{NBS})	From values of $2e/h$ (μV_{NBS})	Direct volt transfers (μV_{NBS})	From values of $2e/h$ (μV_{NBS})
BIPM	-0.17 ± 0.14[c]	-0.17 ± 0.14[c]		-0.28 ± 0.14		-0.22 ± 0.14	
NRC	-0.07 ± 0.14					-0.49 ± 0.14	
NPL	0.52 ± 0.14		1.00 ± 0.81	1.13 ± 0.14	1.16 ± 0.21	1.07 ± 0.14	1.15 ± 0.21
NSL	-0.17 ± 0.14		0.24 ± 0.16	0.45 ± 0.14	0.44 ± 0.11	0.38 ± 0.20	0.60 ± 0.11
PTB	-0.43 ± 0.14			0.09 ± 0.20	0.23 ± 0.40[d]	-0.05 ± 0.20[e]	0.33 ± 0.06

[a] Ref. [1.3].

[b] Note that all V_{LAB} differ from each other and V absolute by a few ppm at most. Thus, small volt differences such as are given in this table are for all practical purposes the same whether expressed in terms of any V_{LAB} or V.

[c] These two values are from the same transfer. The procedures used for the NBS-BIPM 1970 triennial volt intercomparison were the same as those used for the 1971 and 1972 LAB-NBS direct volt transfers.

[d] The PTB $2e/h$ measurement carried out in the Fall of 1970 has been used for the 1971 calculations.

[e] The 1972 direct PTB-BIPM transfer (table 2.1, footnote b) and the 1972 direct NBS-BIPM transfer (this column, first line) imply $V_{PTB} - V_{NBS} = (0.09 \pm 0.20)\ \mu V$.

ries throughout the world, and our improved knowledge of the interrelationships among the as-maintained electrical units of the various countries, the NBS units no longer have the unique position they held in the 1969 analysis of Taylor et al. [0.1]. We shall therefore express all electrical units in the present analysis in terms of BIPM units.

For the purposes of our least-squares adjustment, we shall *define* the 1 January 1969 BIPM unit of voltage as that Josephson step voltage corresponding to an irradiation frequency of 483594.000 GHz. This implies that

$$2e/h = 483594.000\ \text{GHz}/V_{BI69}, \tag{2.1}$$

exactly, where V_{BI69} is the *defined* 1 January 1969 BIPM as-maintained volt. It should be noted that the symbol V_{69-BI} is commonly used to indicate the present unit of voltage as-maintained by the BIPM on the basis of the 1 January 1969 change and, because of the drifts in the standard cells used to maintain it, is a time-dependent unit. The CCE Statement E–72 gives what is believed to be the equivalent Josephson frequency for V_{69-BI} on 1 January 1969. The symbol V_{BI69} as used in this paper represents our *defined* value of V_{69-BI} on 1 January 1969.

With the definition given in eq (2.1), it is necessary to determine from the existing experimental data the magnitude of the unit of voltage which is realized at BIPM and the various national laboratories with groups of standard cells. Since the data of tables 1.1 and 2.2 clearly indicate that the volt maintained at BIPM and the volts maintained at the national laboratories exhibit drifts of the order of a few parts in 10^7 per year, we shall assume that there is a linear dependence on time and write

$$V_i = V_{BI69}(1 + a_i + b_i t), \tag{2.2}$$

where t is the time measured in years from 1 January 1969, and the subscript refers to a specific laboratory. It should be emphasized, however, that although this drifting process was no doubt occurring prior to 1969, there is no experimental evidence and therefore assurance that the drift rates are constant over long periods, for example, a decade. Measurements such as the absolute ampere, proton gyromagnetic ratio, and Faraday have just been too imprecise to indicate parts in 10^7 changes in the various as-maintained units of voltage.

On the basis of the post 1 January 1969 linear drift model, eq (2.2), voltage comparisons between two laboratories yield a relation of the form

$$V_i - V_j = a_i - a_j + (b_i - b_j)t, \tag{2.3}$$

while measurements of $2e/h$ at laboratory i in terms of V_i and at time t provide relations like

$$(2e/h)_{i,t} = E(1 + a_i + b_i t), \tag{2.4}$$

where $E = 483594.000\ \text{GHz}/V_{BI69}$.

A least-squares analysis using eqs (2.2) to (2.4), the eleven $2e/h$ measurements of table 1.1, and the thirteen volt differences resulting from the 1970 BIPM triennial intercomparisons and 1971 and 1972 direct volt transfers (tables 2.1 and 2.2, as summarized in table 2.3 with the appropriate dates), yields

$$V_{BIPM} = V_{BI69} + [-0.026(185) - 0.365(68)t]\ \mu V, \tag{2.5a}$$

$$V_{NBS} = V_{BI69} + [-0.047(153) - 0.317(53)t]\ \mu V, \tag{2.5b}$$

$$V_{NPL} = V_{BI69} + [+0.448(273) - 0.113(100)t]\ \mu V, \tag{2.5c}$$

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TABLE 2.3. Summary of volt intercomparison data used in least squares analysis of $2e/h$ data [a]

1970 triennial intercomparison	Assumed exact mean time of measurements
$V_{NBS} - V_{BIPM} = 0.17(14)\ \mu V$	1970.09
$V_{NPL} - V_{BIPM} = 0.69(14)\ \mu V$	1970.09
$V_{NSL} - V_{BIPM} = 0.00(14)\ \mu V$	1970.09
$V_{PTB} - V_{BIPM} = -0.26(14)\ \mu V$	1970.09
Direct volt transfers	
$V_{BIPM} - V_{NBS} = -0.28(14)\ \mu V$	1971.92
$V_{BIPM} - V_{NBS} = -0.22(14)\ \mu V$	1972.37
$V_{NPL} - V_{NBS} = 1.13(14)\ \mu V$	1971.58
$V_{NPL} - V_{NBS} = 1.07(14)\ \mu V$	1972.24
$V_{NSL} - V_{NBS} = 0.45(14)\ \mu V$	1971.46
$V_{NSL} - V_{NBS} = 0.38(20)\ \mu V$	1972.26
$V_{PTB} - V_{NBS} = 0.09(20)\ \mu V$	1971.65
$V_{PTB} - V_{NBS} = -0.05(20)\ \mu V$	1972.37
$V_{PTB} - V_{BIPM} = 0.31(14)\ \mu V$	1972.65

[a]Tables 2.1 and 2.2, and ref. [1.3].

$$V_{NSL} = V_{BI69} + [-0.210(172) - 0.099(68)t]\ \mu V, \tag{2.5d}$$

$$V_{PTB} = V_{BI69} + [-0.569(254) - 0.081(77)t]\ \mu V. \tag{2.5e}$$

For this analysis, χ^2 is 13.66 for 24 − 10 = 14 degrees of freedom. This indicates that the uncertainties assigned the volt transfers, which are not statistically well defined, are reasonable. (A graphical representation of the $2e/h$ data used and the results of the least-squares analysis is given in fig. 1.)

It should be remembered that all of the uncertainties quoted in eqs (2.5) are correlated. The actual uncertainties in V_i are given by the following expressions:

$$\sigma^2_{BIPM} = (0.0342 - 0.0233t + 0.0046t^2)\ (\text{ppm})^2, \tag{2.6a}$$

$$\sigma^2_{NBS} = (0.0234 - 0.0160t + 0.0028t^2)\ (\text{ppm})^2, \tag{2.6b}$$

$$\sigma^2_{NPL} = (0.0746 - 0.527t + 0.0101t^2)\ (\text{ppm})^2, \tag{2.6c}$$

$$\sigma^2_{NSL} = (0.0297 - 0.0226t + 0.0047t^2)\ (\text{ppm})^2, \tag{2.6d}$$

$$\sigma^2_{PTB} = (0.0643 - 0.0387t + 0.0059t^2)\ (\text{ppm})^2. \tag{2.6e}$$

The uncertainties in V_i corresponding to the time period 1971–72 are of the order of 0.05–0.10 ppm, reflecting the fact that this is the period of the most precise measurements.

Equation (2.5a) implies that on 1 January 1969, the actual BIPM as-maintained unit of voltage was (0.026 ± 0.185) μV less than V_{BI69} as defined by eq (2.1), that it has been exhibiting a drift of −0.37 μV/year, and that it corresponded to a Josephson frequency of 483593.987(90) GHz (0.19 ppm). (Note that as would be expected, this frequency is quite consistent with the CCE statement.) As indicated previously, it is impossible to state with any degree of certainty that such a drift did not exist during the decade prior to 1 January 1969. While the proton gyromagnetic ratio (γ_P) measurements at NBS from 1960 to 1967 [0.1] would appear to rule out such a large drift, the lack of sufficiently precise dimensional measurements of the solenoid used in those experiments prevents an unequivocal statement. We shall therefore assume throughout the present work that prior to 1 January 1969, the BIPM unit of voltage was essentially constant. Or in other words, that any change in the BIPM unit of voltage prior to this date was negligibly small compared with the uncertainties in the experiments carried out during this period that required a unit of voltage and that will be considered for inclusion in our adjustment. (Further motivation for this approach will be given in section II.A.4 where we discuss the relationship between the BIPM ohm and the absolute (SI) ohm.) For such experiments, we convert to V_{BIPM} by linearly interpolating between triennial intercomparisons and assuming a ± 0.14 μV uncertainty for the interpolated volt difference. To finally convert to V_{BI69} will of course require taking into account the 11 ppm 1 January 1969 redefinition of V_{BIPM} (table 2.1) and the (0.026 ± 0.185) ppm correction implied by eq (2.5a) and discussed above.[6] To convert experiments carried out after 1 January 1969 to V_{BI69}, we need only use eq (2.5).

3. Speed of Light in Vacuum, c

All past determinations of c have been rendered obsolete by Evenson et al.'s [3.1] recent measurement of the frequency of a He-Ne laser stabilized on the P(7) line of the ν_3 absorption band of methane (3.39 μm, 88 THz). Through a chain of frequency comparisons of stabilized laser oscillators, Evenson et al. compared the methane absorption line frequency with the frequency of the cesium clock definition of the second. The frequency of the 3.39 μm methane line is thereby established as:

$$\nu(CH_4) = 88376181627(50)\ \text{kHz}. \tag{3.1}$$

The relative standard deviation of the measurement is thus less than 6×10^{-10}. The wavelength of this transition has been measured by Barger and Hall [3.2], by Giacomo [3.3], and by Baird et al. [3.4]. The accuracy of these wavelength determinations is limited

[6] The slight correlation between experiments introduced by including the 0.185 ppm uncertainty in each is entirely negligible.

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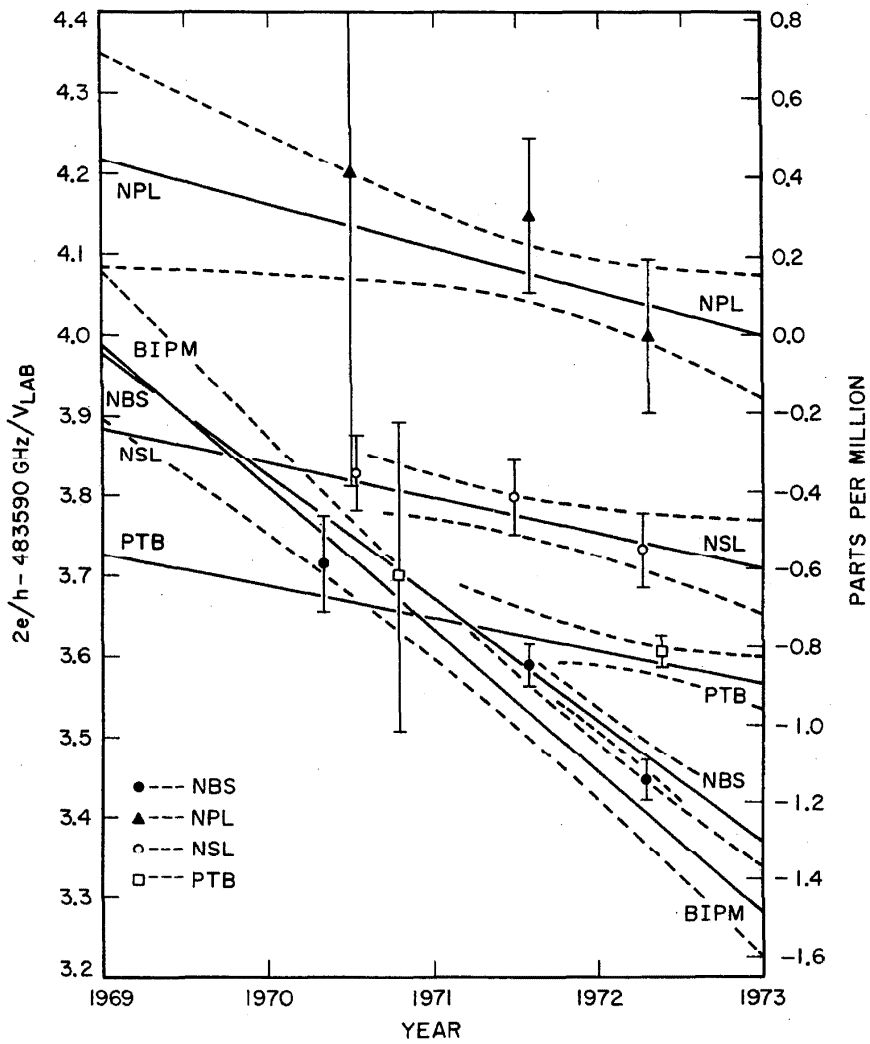

FIGURE. 1. Plot showing the $2e/h$ measurements (points with error bars) used in the least-squares analysis of the post 1 January 1969 drift rates of the BIPM, NBS, NPL, NSL, and PTB as-maintained units of voltage; and the results of the analysis: The linear drift-rate curves, eq (2.5) (full lines); and their uncertainties, eq (2.6) (dotted lines). For clarity, the latter are not shown completely for every laboratory.

not by the measurement process itself but by the uncertainty in the precise definition of the metre in terms of the krypton wavelength. The Kr line is known to be asymmetric and has been analyzed in terms of a satellite line of relative intensity 0.06 displaced 0.008 cm^{-1} or 0.63 half-widths toward the red [3.2]. This leads to a variation of 8.3 parts in 10^9 in the numerical value of a measured wavelength depending on whether the center of gravity or the peak of the line is understood as defining the metre.

Barger and Hall [3.2] at NBS Boulder, using the center of gravity definition, find

$$\lambda(CH_4) = 3392231.376(12) \text{ pm } (0.0035 \text{ ppm}). \quad (3.2)$$

If the peak definition were used, this wavelength would be increased by 0.028 pm to 3392231.404 pm.

Giacomo [3.3] determined the methane wavelength using Michelson's interferometer at BIPM and gives a value

$$\lambda(CH_4) = 3392231.376(8) \text{ pm } (0.0024 \text{ ppm}). \quad (3.3)$$

The path length of the interferometer was such that this measurement corresponds closely to using the center of gravity definition of the krypton line.

Baird, Smith, and Berger at NRC [3.4] found a value

$$\lambda(CH_4) = 3392231.40(2) \text{ pm } (0.0061 \text{ ppm}). \quad (3.4)$$

This value is actually in better agreement with the previous two than appears superficially because it corresponds more nearly to a krypton wavelength midway between the peak and the center of gravity

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definitions. More than half of the difference between eq (3.4) and eq (3.2) or eq (3.3) is therefore ascribable to the difference in the krypton standard; the inherent agreement among these three measurements is more nearly of the order of 0.01 pm or 3 parts in 10^9.

On the basis of the excellent accord among these and other measurements of stabilized laser wavelengths, the Comité Consultatif pour la Définition du Mètre (CCDM) of the CIPM recommended (at their meeting in June 1973) [3.5] the use of the values

$$\lambda(CH_4) = 3392231.40 \text{ pm}, \tag{3.5a}$$

$$\lambda(^{127}I) = 632991.399 \text{ pm}, \tag{3.5b}$$

respectively, for the wavelengths in vacuum of He–Ne lasers stabilized by the $P(7)$ line of the ν_3 band of methane, and the component i of the $R(127)$, 11-5 band of iodine-127. The wavelengths of these radiations are estimated to have the values stated to within 4×10^{-9} in relative value, and this uncertainty is essentially due to the present indeterminacy in the practical realization of the metre.

If the recommended wavelength given in eq (3.5a) is combined with the accurately measured frequency of eq (3.1), one then finds $c = \lambda\nu = 299792458.33$ m/s, with an uncertainty of $\pm$ 1.2 m/s, arising from the uncertainty in the definition of the metre. (The standard deviation based on the experimental uncertainties of the data is 0.6 m/s.) On this basis, the CCDM recommended the value [3.5]

$$c = (299792458 \pm 1.2) \text{ m/s } (0.004 \text{ ppm}). \tag{3.6}$$

Without intending to prejudge any future redefinition of the metre or the second, the CCDM suggested that any such redefinitions should attempt to retain this value provided that the data upon which it is based are not subsequently proved to be in error.

The present least-squares analysis was completed prior to the CCDM meeting and utilized the value [3.6]

$$c = (299792456.2 \pm 1.1) \text{ m/s } (0.0035 \text{ ppm}). \tag{3.7}$$

The difference between this value and eq (3.6) is 0.006 ppm and is entirely negligible compared with the uncertainties of any experimental data involving the speed of light. In our final recommended set of constants we give the value of eq (3.6). None of the other quantities in that table would be significantly altered by the change.

The recommended value given in eq (3.6) is in agreement with other recent independent determinations of the speed of light. Baird et al. [3.7] measured the wavelengths of various CO_2 laser lines in the 9 μm and 10 μm bands with a relative accuracy of approximately 2×10^{-8}. Evenson et al. [3.1], as a part of the chain of frequency measurements from cesium to methane, determined the frequencies of the $R[30]$ transition at 10.18 μm and the $R[10]$ transition at 9.33 μm. These independent wavelength and frequency measurements are then tied together by the accurate frequency difference measurements of the CO_2 bands by Bridges and Chang [3.8] and lead to the value [3.7]

$$c = 299792460(6) \text{ m/s } (0.02 \text{ ppm}). \tag{3.8}$$

Since the uncertainty component from the wavelength measurement is 30 times larger than that from the frequency measurement, eq (3.8) is essentially stochastically independent of eq (3.6).

These measurements are also supported by the value of c reported by Bay, Luther, and White [3.9] using a completely different technique. These workers determined the ratio of the sum and difference frequencies interferometrically of an absorption stabilized He–Ne laser oscillating at 633 nm (474 THz) modulated by a microwave frequency. Hence, they determined the frequency of the laser in terms of the frequency of the microwaves. Combining this with the known wavelength of the laser they obtained

$$c = 299792462(18) \text{ m/s } (0.06 \text{ ppm}). \tag{3.9}$$

These new values of c are all consistent with, but one or two orders of magnitude more accurate than, the previously accepted value [0.1] obtained by Froome in 1957 using microwave interferometry [3.10]:

$$c = 299792500(100) \text{ m/s } (0.33 \text{ ppm}), \tag{3.10}$$

as well as with other measurements [0.1] of comparable accuracy carried out during the past decade.

4. Ratio of BIPM As-Maintained Ohm to Absolute Ohm

As part of our adjustment it is necessary to include the relationship between the absolute electrical units of the Système Internationale d'Unités and the maintained standards against which all of the measurements of interest have actually been made. Since we intend to carry out the present adjustment in terms of V_{BI69} as defined by eq (2.1), we will need to express all quantities used in the present work which require electrical units in terms of 1 January 1969 BIPM units. We shall denote the 1 January 1969 BIPM ohm as realized through standard resistors by the symbol Ω_{BI69}. The required ohm ratio is therefore Ω_{BI69}/Ω. As explained in sections II.A.1 and 2, we have reserved the subscript BI69 to apply *only* to the date 1 January 1969. As applied to the volt, the subscript BI69 has the more specific meaning as defined in eq (2.1). The subscript BIPM or in general, LAB (i.e., NBS, NPL, etc.), on a particular electrical unit means the as-maintained value of that unit at the time of the measurement under consideration.

A value for the quantity Ω_{BI69}/Ω may best be obtained from the measurements carried out at NSL

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using the Thompson-Lampard calculable capacitor [4.1] at the time of the 1964, 1967, and 1970 BIPM triennial international intercomparisons. Thompson at NSL reports [4.2, 4.3]:

$$1964: c_0^2\,\Omega_{NSL}/c^2\,\Omega = 1 - (3.58 \pm 0.06) \times 10^{-6}, \tag{4.1a}$$

$$1967: c_0^2\,\Omega_{NSL}/c^2\,\Omega = 1 - (3.80 \pm 0.06) \times 10^{-6}, \tag{4.1b}$$

$$1970: c_0^2\,\Omega_{NSL}/c^2\,\Omega = 1 - (0.00 \pm 0.06) \times 10^{-6}. \tag{4.1c}$$

The explicit dependence of the measurements on the speed of light has been shown since Thompson used the Froome result for c_0, eq (3.10). The quoted uncertainty is statistical only; the total uncertainty including allowances for systematic effects is 0.2 ppm [4.3]. (The large apparent shift in the NSL ohm in 1970 is due to its 1969 redefinition; see table 2.1.)

Using the 1964, 1967, and 1970 BIPM triennial intercomparison results for the differences between the NSL and BIPM as-maintained ohms yields

$$1964: c_0^2\,\Omega_{BIPM}/c^2\,\Omega = 1 - (0.03 \pm 0.10) \times 10^{-6}, \tag{4.2a}$$

$$1967: c_0^2\,\Omega_{BIPM}/c^2\,\Omega = 1 - (0.17 \pm 0.10) \times 10^{-6}, \tag{4.2b}$$

$$1970: c_0^2\,\Omega_{BIPM}/c^2\,\Omega = 1 - (0.29 \pm 0.10) \times 10^{-6}, \tag{4.2c}$$

where we have included an additional 0.08 ppm uncertainty (assumed random) for the transfer between, and the intercomparison measurements at, NSL and BIPM. Since there was no redefinition of the BIPM ohm on 1 January 1969, we may, without further correction, fit a straight line to the data of eq (4.2) since they clearly indicate a simple linear drift of the BIPM ohm. Measuring time in years from 1 January 1969, and using the central dates of the triennial intercomparisons as the precise times to be associated with eq (4.2), we obtain [after substituting the value of c given in eq (3.7)]:

$$\Omega_{BIPM} = \Omega + [-0.538(5) - 0.043(1)t]\,\mu\Omega, \tag{4.3a}$$

with

$$\sigma^2{}_{BIPM} = (20.76 + 8.185t + 2.147t^2) \times 10^{-6}\ (\text{ppm})^2. \tag{4.3b}$$

For this analysis, $\chi^2 = 0.0039$ for one degree of freedom. Such a low value would seem rather fortuitous. Note that the 0.19 ppm systematic uncertainty in the calculable capacitor measurements must be added separately to the uncertainty given in eq (4.3b). The final result for Ω_{B169}/Ω [setting $t = 0$ in eq (4.3a)] is thus

$$\Omega_{B169}/\Omega = 0.99999946(19)\ (0.19\ \text{ppm}). \tag{4.4}$$

Although eq (4.3a) indicates that the BIPM ohm has been decreasing at the rate of $-0.043\ \mu\Omega$/year since at least 1964, we shall ignore it for the entire period prior to 1 January 1969. The reason is that all experiments of interest carried out during this period will involve the BIPM volt as well. As was discussed in detail in section II.A.2, no correction for a possible drift in V_{BIPM} prior to this date will be applied. Since if anything, V_{BIPM} was probably decreasing during this period, and the quantity which really enters the relevant experiments is the BIPM as-maintained ampere, $A_{BIPM} = V_{BIPM}/\Omega_{BIPM}$, it would be wrong to correct one without correcting the other. That is, A_{BIPM} has very likely been more stable than either V_{BIPM} or Ω_{BIPM}. We shall therefore reexpress in terms of A_{BIPM} the results of pre 1 January 1969 experiments carried out in terms of A_{LAB} by linearly interpolating between the appropriate BIPM triennial intercomparisons. An uncertainty of 0.16 μA will be assumed for the interpolated ampere difference (0.14 μV for the volt difference and 0.08 $\mu\Omega$ for the ohm difference). To finally convert to A_{B169} will, of course, require taking into account the 11 ppm and 0.026 ppm corrections to V_{BIPM} discussed in section II.A.2.

Since the drift in V_{BIPM} must be taken into account for post 1 January 1969 experiments, we must also do the same for Ω_{BIPM}. As we shall see, this means knowing the (time dependent) differences $\Omega_{NBS} - \Omega_{B169}$ and $\Omega_{NPL} - \Omega_{B169}$ since these are the only two laboratories with relevant experiments. Assuming linear drifts, using the results of the 1964, 1967, and 1970 triennial intercomparisons, measuring time in years from 1 January 1969, and taking into account the 3.7 $\mu\Omega$ redefinition of Ω_{NPL} on this same date, we find:

$$\Omega_{NBS} = \Omega_{B169} + [-0.048(49) + 0.046(16)t]\,\mu\Omega, \tag{4.5a}$$

$$\sigma^2{}_{NBS} = (24.26 + 9.563t + 2.508t^2) \times 10^{-4}\ (\text{ppm})^2, \tag{4.5b}$$

$$\Omega_{NPL} = \Omega_{B169} + [0.271(33) + 0.018(11)t]\,\mu\Omega, \tag{4.5c}$$

$$\sigma^2{}_{NPL} = (11.16 + 4.402t + 1.154t^2) \times 10^{-4}\ (\text{ppm})^2. \tag{4.5d}$$

χ^2 is 0.71 and 0.33, respectively, one degree of freedom, assuming an *a priori* assigned uncertainty of 0.08 $\mu\Omega$ for each triennial intercomparison.

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5. Acceleration Due to Gravity, g

The acceleration due to gravity is needed at four places: The former site of the NBS current balance and Pellat electrodynamometer; the sites of the NPL current balance and high field γ_p experiments; and the site of the Kharkov, U.S.S.R., high field γ_p experiment. As discussed in ref. [0.1] the required values may be obtained from g(CB), g(BFS), and g_P (Kharkov), where CB stands for the Commerce Department Building; BFS means the British Fundamental Station; and g_P (Kharkov) is the value of g at Kharkov on the Potsdam System.

In the present work, we shall use the g values of IGSN71, the International Gravity Standardization Net 1971 [5.1]. This network, developed under the auspices of the International Union of Geodesy and Geophysics, is a least-squares adjusted self-consistent worldwide gravity net based on 25,000 absolute, gravimeter and pendulum measurements. It provides gravity values with an uncertainty of less than 0.1 mGal over the gravity range of the earth (1 mGal = 10^{-5} m/s^2 $\approx$ 1 ppm in g). Furthermore, these values include the so called Honkasalo correction, that is, the IGSN71 g values are the average values that would be measured at a particular site in a continuous experiment extending over a period of time long enough to completely cover the lunar and solar cycles. The absolute values used as input data for the net include measurements by Cook [5.2], Tate [5.3], Faller and Hammond [5.4], and Sakuma [5.5]. The results are:

$$g\,(\mathrm{CB}) = 980104.30 \pm 0.02\ \mathrm{mGal}\ (0.02\ \mathrm{ppm}), \tag{5.1a}$$

$$g\,(\mathrm{BFS}) = 981181.77 \pm 0.02\ \mathrm{mGal}\ (0.02\ \mathrm{ppm}). \tag{5.1b}$$

Unfortunately, the IGSN71 adjustment does not include any data from the Soviet Union. Therefore we are forced to obtain g(Kharkov) from the older data for the difference between the values of g at Potsdam and at Kharkov, and to assume this *difference* is reasonably accurate. Since the IGSN71 gives −14.0 mGal as the best correction to the Potsdam System at Potsdam, we shall take

$$g(\mathrm{KhGNIIM}) = g_P(\mathrm{Kharkov}) - (14.0 \pm 1.0)\ \mathrm{mGal}\ (1\ \mathrm{ppm}), \tag{5.1c}$$

where the 1 ppm uncertainty is assigned somewhat arbitrarily to take into account possible errors in the difference between g at Potsdam and Kharkov. Although this uncertainty is thus much larger than the uncertainty in those values for which we have modern determinations, we may still use eq (5.1c) to include the Kharkov γ_p'(high field) measurement in our adjustment since its uncertainty is essentially uncorrelated with any other sources of uncertainty.

6. g-Factors of the Free Electron and Muon, g_e and g_μ

The magnetic moment of the electron in Bohr magnetons enters our least-squares adjustment in two ways: As an auxiliary constant in the form of the free electron g-factor, and as a stochastic input datum in the form of the electron magnetic moment anomaly, where it provides a determination of the fine-structure constant. We shall postpone the discussion of the theoretical interpretation to section II.C.19, treating the data here only as empirical values which are theory-independent.

The free electron g-factor, $g_e = 2\mu_e/\mu_B$, where μ_e is the magnetic moment of the electron and $\mu_B = e\hbar/2m_e$ is the Bohr magneton, follows directly from the recent experimental determination by Wesley and Rich [6.1] of the anomalous magnetic moment of the electron, a_e:

$$g_e/2 = \mu_e/\mu_B = 1 + a_e = 1.0011596567(35)\ (0.0035\ \mathrm{ppm}). \tag{6.1}$$

This is actually the revised result due to Granger and Ford [6.2]. (The original Wesley-Rich result was a_e = 0.0011596577(35).) These workers reconsidered electron spin motion in a magnetic mirror trap of the sort used in the $g-2$ experiments at the University of Michigan. Their new approach has also led to a major correction to the lower accuracy Wilkinson-Crane [6.3] value for a_e obtained in the early 1960's and thus to the resolution of the discrepancy between this value and that of Wesley and Rich. Significant validity to the Granger-Ford theoretical analysis is thereby added. It is also reassuring that the Wesley-Rich value of a_e is in good agreement with the best present theoretical result as given recently by Kinoshita and Cvitanovic [6.4] (to be discussed in sec. II.C.19).

The free muon g-factor $g_\mu/2 = \mu_\mu(e\hbar/2m_\mu)^{-1}$, where μ_μ and m_μ are respectively the magnetic moment and rest mass of the muon, will later be required for calculating a value of the ratio m_μ/m_e. We adopt the value

$$g_\mu/2 = 1 + a_\mu = 1.00116616(31)\ (0.31\ \mathrm{ppm}), \tag{6.2}$$

which follows directly from the CERN muon storage ring determination of a_μ [6.5]. This in turn is in *agreement with the present theoretical result* (to be discussed in sec. II.C.19). The 0.31 ppm uncertainty in g_μ is sufficiently small compared with the uncertainties assigned the other quantities required to calculate m_μ/m_e that it may be taken as an auxiliary constant. Similarly, the free electron g-factor, eq (6.1), may also be taken as exactly known as far as our adjustment is concerned.

7. Magnetic Moment of the Proton in Units of the Bohr Magneton, μ_p/μ_B

A value for μ_p/μ_B may best be derived from the

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hydrogen maser measurement of $g_j(\mathrm{H})/g_p(\mathrm{H})$, the ratio of the electron and proton g factors in the ground or $1S$ state of hydrogen (obtained at the same magnetic field), by Winkler and co-workers [7.1]. Their result may be taken to be

$$g_j(\mathrm{H})/g_p(\mathrm{H}) = 658.2107063(66)\ (0.010\ \mathrm{ppm}). \quad (7.1)$$

This ratio must now be corrected to the ratio of the free electron and proton g factors in order to obtain μ_e/μ_p and subsequently μ_p/μ_B. To do this we use the theory of Grotch and Hegstrom [7.2] which has been substantiated by the good agreement found between the theoretical and experimental values for the hydrogen-deuterium g factor ratio [7.3]. (The calculations of other workers also confirm the Grotch-Hegstrom corrections [7.4, 7.5].) Although such accuracy is not really required, we anticipate the results later to be obtained and evaluate the Grotch-Hegstrom theory using $\alpha^{-1} = 137.036$, $m_p/m_e = 1836.15$, and $\mu_p/\mu_N - 1 = a_p = 1.7928$, and obtain

$$g_j(\mathrm{H})/g_e = 1 - 17.705 \times 10^{-6}, \quad (7.2a)$$

$$g_p(\mathrm{H})/g_p = 1 - 17.733 \times 10^{-6}. \quad (7.2b)$$

These correction factors may be in error by several parts in 10^9 because of the neglect of uncalculated terms of order $(Z\alpha)^4 \approx 3 \times 10^{-9}$ [7.2]. However, to the accuracy presently needed, they may be considered to be exact. Applying these corrections to eq (7.1) yields

$$g_e/g_p = \mu_e/\mu_p = 658.2106880(66)\ (0.01\ \mathrm{ppm}). \quad (7.3)$$

Finally, we obtain μ_p/μ_B by combining eq (7.3) with the Wesley-Rich result, eq (6.1), since $\mu_p/\mu_B = (\mu_e/\mu_B)/(\mu_e/\mu_p)$:

$$\mu_p/\mu_B = 0.001521032209(16)\ (0.011\ \mathrm{ppm}). \quad (7.4)$$

For purposes of our least-squares adjustment, both μ_e/μ_p and μ_p/μ_B, eqs (7.3) and (7.4), may be taken as exactly known.

8. Magnetic Moment of the Proton in H_2O in Units of the Bohr Magneton, μ_p'/μ_B

Since many measurements of interest utilize H_2O NMR probes as the hydrogen or proton containing sample, a value of μ_p'/μ_B (where the prime means for protons in a spherical sample of pure H_2O) is required to incorporate them in an adjustment. A value for this quantity may best be obtained from the Lambe-Dicke [8.1] microwave absorption measurement of the ratio $g_j(\mathrm{H})/g_p(\mathrm{H_2O})$ in hydrogen:

$$g_j(\mathrm{H})/g_p(\mathrm{H_2O}) = 658.2159088(436)\ (0.066\ \mathrm{ppm}). \quad (8.1)$$

Correcting $g_j(\mathrm{H})$ for bound state effects using eq (7.2a) yields

$$g_e/g_p(\mathrm{H_2O}) = \mu_e/\mu_p' = 658.2275628(436)\ (0.066\ \mathrm{ppm}). \quad (8.2)$$

As in the previous section, μ_p'/μ_B may finally be obtained by combining eq (8.2) with the Wesley-Rich value of μ_e/μ_B, eq (6.1). We find

$$\mu_p'/\mu_B = 0.001520993215(100)\ (0.066\ \mathrm{ppm}). \quad (8.3)$$

This result is well supported by the value obtained by Klein [8.2] using a rather different method. His result may be taken to be

$$\mu_p'/\mu_B = 0.00152099362(74)\ (0.49\ \mathrm{ppm}). \quad (8.4)$$

The diamagnetic shielding correction for protons in a spherical sample of pure H_2O may be obtained by combining the Winkler et al. and Lambe-Dicke results, eqs (7.3) and (8.2). The result is

$$\sigma(\mathrm{H_2O}) = (25.637 \pm 0.067)\mathrm{ppm}. \quad (8.5)$$

As far as our least-squares adjustment is concerned, both μ_p'/μ_B and $\sigma(\mathrm{H_2O})$, eqs (8.3) and (8.5) may be assumed to be exactly known.[7]

9. Atomic Masses and Mass Ratios

We use as required the relative atomic masses of the nuclides to be published shortly by Wapstra, Gove, and Bos [9.1]. (See table 9.1.) This new compilation will replace the previously recommended set published in 1971 [9.2]. That a revision is necessary in so short a time is due primarily to the very accurate measurements of Smith [9.3] which only became available after the bulk of the work for the 1971 mass evaluation was completed. (A few revised nuclidic masses based on the Smith data were in fact given in an appendix to the Wapstra-Gove paper.)

In addition to the relative atomic masses of various nuclides, we require values for certain mass ratios. To calculate these we first anticipate the result of our adjustment and adopt the value $\mu_p'/\mu_N = 2.792774$ in order to compute the ratio $m_p/m_e = M_p/M_e$. (Throughout, capital letters will be used for relative atomic masses and lower case letters for absolute masses.) Noting that $m_p/m_e = (\mu_p'/\mu_N)/(\mu_p'/\mu_B)$, and using eq (8.3), yields

$$m_p/m_e = 1836.152, \quad (9.1)$$

[7] Throughout the present work, we have neglected the effect of temperature on those experiments utilizing H_2O NMR (and similar) probes since the temperature dependence of the diamagnetic shielding correction, $\sigma(H_2O)$, is only $\approx 10^{-8}$/°C. (See ref. [8.3].) Most of the measurements of interest have been carried out at or near room temperature, and furthermore, most are of insufficient accuracy to warrant correction. However, this situation may change in the future with the advent of parts in 10^8 values of the proton gyromagnetic ratio.

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TABLE 9.1 Values of various relative atomic masses and abundance ratios used in this work[a]

Nuclide	Relative atomic mass[b] ($m(^AX)/m_u$)	Relative abundance[c]	Uncertainty in mass (ppm)
n	1.008665012(37)		0.037
^1H	1.007825036(11)	0.9998508	0.011
^2H	2.014101795(21)	0.0001492(70)	0.010
^3H	3.016049302(33)		0.011
^3He	3.016029307(33)		0.011
^4He	4.002603267(48)		0.012
^{12}C	12.000000000	0.988930	(by definition)
^{13}C	13.00335488(23)	0.011070(21)	0.018
^{16}O	15.994914464(55)	0.997587	0.0034
^{17}O	16.99913237(96)	0.000374(6)	0.056
^{18}O	17.99915900(22)	0.002039(20)	0.012
^{20}Ne	19.99243901(34)		0.017
^{28}Si	27.97692825(78)	0.922027	0.028
^{29}Si	28.97649639(93)	0.047030(191)	0.032
^{30}Si	29.9737717(11)	0.030943(184)	0.036
^{40}Ar	39.96238209(56)		0.014
^{40}Ca	39.96258992(87)	0.969668	0.022
^{42}Ca	41.9586214(20)	0.006400(100)	0.047
^{43}Ca	42.9587702(20)	0.001450(40)	0.046
^{44}Ca	43.9554851(20)	0.020599(400)	0.046
^{46}Ca	45.9536865(45)	0.000033(1)	0.097
^{48}Ca	47.9525262(56)	0.001850(20)	0.12
^{107}Ag	106.9050903(69)	0.518297	0.065
^{109}Ag	108.9047547(48)	0.481703(113)	0.044
^{127}I	126.9044766(48)		0.038

[a] Ref. [9.1]. [b] Neutral atom. [c] Ref. [9.5]; and see text.

a result which should be reliable to at least 1 ppm.

If the relative atomic mass of the neutral hydrogen atom is $M_{^1H}$, and if its binding energy is included ($\simeq -\alpha^2 m_e c^2/2$), we can write

$$M_p = M_{^1H}\left[1 + (1-\alpha^2/2)\frac{m_e}{m_p}\right]^{-1},$$

with an accuracy of 8×10^{-12}. If the relative atomic mass of a nucleus is $M_{^AN}$ and the corresponding relative atomic mass of the neutral atom is $M_{^AX}$ we can also write

$$M_{^AN} = M_{^AX} - M_p\frac{m_e}{m_p}Z + |E_B/c^2|.$$

For hydrogen (and deuterium) the binding energy is $|E_B/c^2| - 13.6$ eV $- 15$ nu, and for helium $|E_B/c^2| - 79.0$ eV $= 85$ nu [9.4]. Using the masses of Wapstra et al., table 9.1, and noting that $M_e = M_p(m_e/m_p)$, and $M_e/M_{^AN} = m_e/m_{^AN}$ we finally obtain

$$M_p = 1.007276470(11)\ (0.011\ \text{ppm}), \quad (9.2a)$$

$$1 + m_e/m_p = 1.000544617, \quad (9.2b)$$

$$1 + m_e/m_d = 1.000272444, \quad (9.2c)$$

$$1 + m_e/m_\alpha = 1.000137093. \quad (9.2d)$$

The 0.011 ppm uncertainty in M_p is due to the 0.011 ppm uncertainty assigned $M_{^1H}$ by Wapstra et al. Based on an assumed 1 ppm uncertainty for m_p/m_e, the uncertainty in the last three numbers is less than 1 in the last digit. i.e., $<1/10^9$. In all four cases these quantities may be assumed to be exactly known as far as our adjustment is concerned. This is also true of the quantity

$$1 + m_e/m_\mu = 1.00483634(3)\ (0.03\ \text{ppm}), \quad (21.7)$$

which will be derived in section II.C.21 and which is given here for completeness.

We have also included in table 9.1 the relative isotopic abundances for those nuclides which must be used to calculate various atomic and molecular weights required in the present work. (These weights are given in table 9.2.) The abundances are taken from ref. [9.5] (the recommended or "A" values) but are normalized so that they sum to unity. The uncertainties assigned the abundances are our own standard deviation estimates and follow from the uncertainties assigned the abundance measurements and their range as given in ref. [9.5]. (Where applicable, we divide the range by 3 in order to obtain a 68% confidence level estimate.) For Si, the uncertainties were calculated as in ref. [17.1]. For Ag, the abundances and their uncertainties

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TABLE 9.2. Values of various relative atomic and molecular weights used in this work[a]

Substance	Symbol	Relative atomic or molecular weight	Uncertainty (ppm)
Hydrogen	H	1.0079752(70)	7.0
Carbon	C	12.011107(21)	1.8
Oxygen	O	15.999377(41)	2.5
Silicon	Si	28.08573(41)	15
Calcium	Ca	40.0769(16)	40
Silver	Ag	107.86833(23)	2.1
Benzoic acid	$C_7H_6O_2$	122.12435(17)	1.4
Oxalic acid dihydrate	$C_2H_2O_4 \cdot 2H_2O$	126.06633(25)	2.0
Calcite	$CaCO_3$	100.0862(16)	16

[a] Calculated from the data of table 9.1.

were calculated from the ratio $^{107}Ag/^{109}Ag$ = 1.07597(49) as derived in ref. [0.1].

10. Rydberg Constant for Infinite Mass, R_∞

In the adjustment of Taylor et al., R_∞ was based equally on (a) the pre-WWII data of Houston (1927) [10.1], Chu (1939) [10.2], and Drinkwater, Richardson, and Williams (1940) [10.3]; and (b) that of Csillag (1966) [10.4, 10.5]. However, since the Taylor et al. review appeared, three new measurements have been completed. Thus, although for the present adjustment we have once again reviewed and revised the pre-WWII data, we shall make no real use of the results. The reason is simply that no matter how the older data are handled, there are many questions and ambiguities which cannot be resolved, for example, intensity anomalies and Doppler broadening. Rather, we believe a much more reliable result may be obtained solely from the modern measurements. These are summarized in table 10.1. (Here, R_∞ was calculated from the equation $R_\infty = R_i(1 + m_e/m_i)$ and the values of $(1 + m_e/m_i)$ given in eq (9.2).) The following comments apply to the results given in this table.

(a) Csillag. The value quoted is our own revision of Csillag's original result, R_D = 109707.4167(28) cm^{-1} (0.026 ppm), statistical uncertainty only [10.5]. The basis of the revision is the inclusion of the Doppler broadening of the Balmer pattern determined from an estimated effective gas temperature for the spectral source used by Csillag. The uncertainty quoted in the table includes a 0.027 ppm statistical component; a 0.02 ppm systematic component arising from the uncertainty in the wavelength of Csillag's ^{198}Hg lamp which was compared against ^{86}Kr by Rowley at NPL [10.9]; 0.02 ppm for the uncertainty in the index of refraction correction for nonstandard air; 0.02 ppm for phase shift error; 0.03 ppm for the effect of overlapping lines; 0.01 ppm for possible stark shifts; and 0.01 ppm to allow for uncertainty in the realization of the metre.

(b) Masui. Masui's original value was 109677.5937(35) cm^{-1} (0.032 ppm), statistical uncertainty only [10.6], and was based on the assumption of theoretical intensities for the Balmer components. The value given in the table is Masui's own recent reevaluation of his original data [10.7]. In this revision, Masui assigned the Hα line $3P-2S$ transitions higher intensity by a factor of 1.46 ± 0.02 than the theoretical values, a choice based on a least-squares fitting of the experimentally observed pattern. Unfortunately, Masui has not published a complete account of his measurements with a discussion of possible systematic errors. To allow for systematic errors one should probably multiply his quoted statistical uncertainty by at least a factor of two. We certainly cannot conclude that Masui's measurement is significantly more accurate than the other experiments of table 10.1.

(c) Kessler, and Kibble et al. Both determinations utilized computer aided deconvolution procedures. The quoted uncertainties, which are those given by the authors, include both random and systematic components.

We shall adopt the simple average of the four measurements given in table 10.1,

$$R_\infty = 109737.3177(83)\ \text{cm}^{-1}\ (0.075\ \text{ppm}), \quad (10.1)$$

for use as an auxiliary constant in our adjustment. We choose not to take a weighted average because we

TABLE 10.1. Summary of modern measurements of the Rydberg constant

Publication date and author	Result (cm^{-1})	Implied value of R_∞ (cm^{-1})	Uncertainty (ppm)
1968, Csillag[a]	R_D= 109707.4169(60)	109737.3060(60)	0.055
1971, Masui[b]	R_H= 109677.5865(45)	109737.3188(45)	0.041[b]
1972, Kessler[c]	R_{He}= 109722.2786(85)	109737.3208(85)	0.077
1972, Kibble et al.[d]	R_D= 109707.4362(77)	109737.3253(77)	0.070

[a] Refs. [10.4, 10.5]. [b] Refs. [10.6, 10.7]. The uncertainty quoted is statistical only; see text. [c] Ref. [10.8]. [d] Ref. [10.9].

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TABLE 11.1. Summary of the more precise data as discussed in sections 1 through 10

Quantity	Units	Value	Uncertainty (ppm)	Eq. No.
$2e/h$	GHz/V_{B169}	483594.000	definition	(2.1)
c	m/s	299792458(1.2)[a]	0.004	(3.6)
Ω_{B169}/Ω		0.99999946(19)	0.19	(4.4)
g(CB)	10^{-5}m/s^2	980104.30(2)	0.02	(5.1a)
g(BFS)	10^{-5}m/s^2	981181.77(2)	0.02	(5.1b)
g(Kharkov)	10^{-5}m/s^2	g_P(Kharkov)−14.0	1.0	(5.1c)
$g_e/2 = \mu_e/\mu_B$		1.0011596567(35)	0.0035	(6.1)
$g_\mu/2$		1.00116616(31)	0.31	(6.2)
μ_e/μ_p		658.2106880(66)	0.010	(7.3)
μ_p/μ_B		0.001521032209(16)	0.011	(7.4)
μ_p'/μ_B		0.001520993215(100)	0.066	(8.3)
$\sigma(H_2O)$	10^{-6}	25.637(67)	0.067[b]	(8.5)
M_μ		1.007276470(11)	0.011	(9.2a)
$1 + m_e/m_p$		1.000544617	<0.001	(9.2b)
$1 + m_e/m_d$		1.000272444	<0.001	(9.2c)
$1 + m_e/m_\alpha$		1.000137093	<0.001	(9.2d)
$1 + m_e/m_\mu$		1.004836323(11)[c]	0.011	
R_∞	m^{-1}	10973731.77(83)	0.075	(10.1)

[a] This is the CCDM recommended value. The value used as an auxiliary constant throughout the present work is 299792456.2 m/s; see section II.A.3.

[b] Uncertainty in $1 + \sigma$ (H_2O).

[c] This is the output value of our final least-squares adjustment. The value used as an auxiliary constant throughout the present work is 1.00483634; see section II.C.21.

believe that basically there is little difference in the reliability of the four values; each measurement has its own peculiar set of difficulties and one is not to be preferred over another. The uncertainty given in eq (10.1) is the statistical standard deviation of the four values rather than the statistical standard deviation of their mean in order to allow for unknown systematic effects in what are comparatively difficult experiments. We also note for purposes of comparison only that (a) the weighted average of the four values is 109737.3168(40) cm^{-1} external consistency, σ_E;[8] (b) the weighted average of the revised pre-WWII data is 109737.3185(120) cm^{-1}, external consistency, σ_E; and (c) deleting the rather internally inconsistent Houston measurements from the revised pre-WWII data gives 109737.3091(60) cm^{-1}, internal consistency, σ_I.

11. Summary of the More Precise Data

Table 11.1 summarizes the more precise data so far discussed. The uncertainties are included for information and comparison purposes only since in most instances, these quantities will be taken as auxiliary constants. The equation numbers used in the text for these quantities are indicated in the column headed "Eq. No.".

[8] Recall that σ_I, the uncertainty determined by internal consistency, is the expected uncertainty in the mean as determined by the *a priori* uncertainties, σ_i, assigned each individual measurement; and that σ_E is the expected uncertainty as determined by how much each individual measurement deviates from the weighted mean in comparison with its a priori uncertainty σ_i. The Birge ratio, R_B, is defined as σ_E/σ_I and is related to χ^2 by $R_B = [\chi^2/\nu]^{1/2}$, where ν is the number of degrees of freedom. The expectation value of χ^2 is ν, and thus of R_B^2, unity. $R_B>1$ generally implies that either the σ_i have been underestimated or that some or all of the data contain systematic errors. Wherever applicable, we shall quote the larger uncertainty.

B. The Less Precise WQED Data

Following Taylor et al., we divide the less precise data into two groups: that which does not require the use of quantum electrodynamic theory for its analysis, hereafter referred to as "without quantum electrodynamic theory" or "WQED" data; and that which does require QED theory for its analysis. While the situation regarding the agreement between QED theory and experiment has now reached the point where QED data may be unequivocally considered for use in an adjustment (see, for example, refs. [19.1, 23.1]), we choose to continue Taylor et al.'s practice of dividing the data in this manner for two reasons. First, it is a convenient way to categorize a rather large amount of information. Second, since many workers in the QED field prefer to use WQED constants when comparing QED theory and experiment, we felt obligated to provide a set of such constants. (It should perhaps be emphasized here that it was not essential to use QED theory for treating the data so far discussed. In every instance, any QED correction was sufficiently small that it could be ignored without materially affecting the output values of our adjustment.)

12. Ratio of BIPM As-Maintained Ampere to Absolute Ampere

The data relevant to the determination of the BIPM as-maintained-ampere-to-absolute-ampere conversion factor, $K = A_{B169}/A$, are summarized in table 12.1. They have been taken in most part from ref. [0.1] but with the following changes and additions:

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TABLE 12.1. Summary of absolute ampere determinations

Publication date, laboratory, and author	Method	A_{LAB}/A	A_{BIPM}/A	$K \equiv A_{BI69}/A$	Uncertainty (ppm)	Eq. No.
1968, NBS Driscoll and Olsen[a]	Pellat balance	1.0000103	1.0000127	1.0000018(97)	9.7	(12.1)
1958, NBS Driscoll and Cutkosky[b]	Current balance	1.0000092	1.0000098	0.9999988(77)	7.7	(12.2)
1965, NPL Vigoureux[c]	Current balance	1.0000171	1.0000098	0.9999988		
1970, NPL Vigoureux and Dupuy[d]	Current balance	1.0000025		1.0000025		
	NPL data averaged in ratio 2:1[e]			1.0000000(55)	5.5	(12.3)

[a] Ref. [12.1]. [b] Ref. [12.2]. [c] Ref. [12.3]. [d] Ref. [12.4]. [e] See text.

(a) 1968 NBS Pellat balance determination. The original data [12.1] were reevaluated with improved precision yielding $A_{NBS}/A = 1.0000094$. In this calculation, the acceleration due to gravity at the site of the balance was taken as 9.80083 m/s². The new IGSN71 value for g(CB), eq (5.1a), implies that g at the site of the balance is actually 9.8008484 m/s². Thus, the above result must be increased by 0.94 ppm to 1.0000103. To convert to BIPM units, we use the result $A_{NBS} - A_{BIPM} = (2.39 \pm 0.16)\ \mu A$ as obtained from the 1967 BIPM intercomparison (18 February central date), since the NBS Pellat measurements were carried out from January to April 1967. The final result in terms of A_{BI69} is obtained using the 11 ppm and (0.026 ± 0.185) ppm volt corrections outlined in sections II.A.2 and 4.[9] The other uncertainty components are as in ref. [0.1], but with no uncertainty assigned g since it is an auxiliary constant.

(b) 1958 NBS current balance determination. The original result [12.2], $A_{NBS}/A = 1.0000083$, has been revised to the value given in the table by including the 0.94 ppm correction implied by the new IGSN71 value of g(CB) (see above). We convert to BIPM units by linearly interpolating between the 1955 and 1957 BIPM intercomparisons since the NBS measurements were carried out in May of 1956 (22 May 1956 mean date). The interpolation yields $A_{NBS} - A_{BIPM} = (-0.55 \pm 0.16)\ \mu A$. The uncertainty given in the table is the RSS of the various components listed in ref. [12.2] (converted from a probable error to a standard deviation[10]), but g is now taken to be an auxiliary constant.

(c) 1965 and 1970 NPL current balance determinations. The October-November 1962 and February-April 1963 series of measurements give, respectively, $A_{NPL}/A = 1.0000135 \pm 1.1$ ppm and $A_{NPL}/A = 1.0000166 \pm 0.8$ ppm [12.3]. (These uncertainties are the statistical standard deviations of the means of the series in contrast to the corresponding uncertainties given in ref. [0.1] which are the statistical standard deviations of the series themselves.) Taking into account the new IGSN71 value of g(BFS), eq (9.1b), requires a −0.73 ppm correction to each; and including the effect of strain, etc, requires a 2.29 ppm correction [12.4]. The value in the table is the weighted mean of the two corrected values. We use $A_{NPL} - A_{BIPM} = (7.25 \pm 0.16)\mu A$ to convert to BIPM units, a value obtained by linearly interpolating between the 1961 and 1964 triennial intercomparisons. (The mean date of the two series of measurements was taken as 7 January 1963.)

The result of the 1970 experiment (carried out December to April 1969–1970) is as given by Vigoureux [12.4] but has been corrected by −0.06 ppm due to the new IGSN71 value of g(BFS). Since this is a post 1 January 1969 measurement, we convert to BI69 units using eqs (2.5c) and (2.6c), and eqs (4.5c) and (4.5d). Taking the mean date of the experiment as 15 February 1970, we find $V_{NPL} - V_{BI69} = (0.32 \pm 0.17)\ \mu V$, $\Omega_{NPL} - \Omega_{BI69} = (0.29 \pm 0.04)\ \mu\Omega$, and thus $A_{NPL} - A_{BI69} = (0.03 \pm 0.17)\ \mu A$. The difference between the 1962/63 and 1969/70 measurements is somewhat surprising in view of the precision of the experiment, but of course, no dimensional measurements were carried out for the 1970 determination. Rather, the dimensions obtained at the time of the earlier measurements were used. A comparison of the calculated and measured differences of the forces exerted by the two coil systems of the balance did indicate that the coil dimensions could not have changed significantly. (Minor improvements involving the beam suspension and scale pans, and a test of the symmetry of the

[9] Although not to be specifically mentioned again, this last procedure will be followed in the remainder of this section and in the next two sections for the pre 1 January 1969 experiments. Similiarly, the 0.185 and 0.16 ppm uncertainties will always be included even if they are not specifically mentioned.

[10] Throughout the present work probable errors (P.E.), that is, 50% confidence level uncertainty estimates, have been converted to standard deviations by multiplying the P.E. by 1.48.

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balance beam, were also carried out.) In view of the fact that no new dimensional measurements were made during 1969–70, we choose not to discard the older result in favor of the new one. On the other hand, the new work should not be ignored entirely since it does represent a significant amount of effort. Thus, we combine the old and new results in the ratio 2:1 to obtain the value given in table 12.1. The quoted uncertainty follows Vigoureux [12.3] with the exception that g is now taken to be an auxiliary constant and the following standard deviation uncertainties have been added to the uncertainty in the ampere ratio: 0.5 ppm for the strain correction [0.1]; 0.4 ppm for temperature [12.4]; and 0.5 ppm for statistical scatter.

It should be noted that we have not included in table 12.1 the result from VNIIM [12.5] (All-Union Scientific Research Institute of Metrology, U.S.S.R.) because investigations are now underway there to clarify the current distribution correction [12.6]. Their present result based on measurements carried out in 1966 is [12.5]

$$A_{IMM}/A = 1.0000166(60)\ (6.0\ \text{ppm}), \qquad (12.5)$$

which implies using the 1967 BIPM triennial intercomparison result

$$A_{BI69}/A = 0.9999967(60)\ (6.0\ \text{ppm}). \qquad (12.6)$$

13. Faraday Constant, F

The relevant values of the Faraday Constant are summarized in table 13.1. The following comments apply.

(a) NBS silver-perchloric acid measurement. The value given in the table for the Craig et al. [13.1] determination is taken from ref. [0.1]: the new ^{107}Ag and ^{109}Ag nuclidic masses (table 9.1) and resulting atomic weight of Ag (table 9.2) leave it unchanged. The nine runs on which the quoted result is based were carried out from January to July, 1958, with a mean date of 18 March 1958. Using this date to interpolate linearly between the 1957 and 1961 BIPM triennial intercomparisons yields $A_{NBS} - A_{BIPM} = (-0.46 \pm 0.16)\ \mu A$, which we use to convert to BIPM units.

(b) NBS benzoic acid and oxalic acid measurements. Marinenko and Taylor [13.2] have coulometricly measured the electrochemical equivalents of benzoic acid ($C_7H_6O_2$) and oxalic acid dihydrate ($C_2H_2O_4 \cdot 2H_2O$). For benzoic acid, they find $E(C_7H_6O_2) = 1.2657155 \times 10^{-6}$ kg/A_{NBS}·s. When this result is combined with the molecular weight for $C_7H_6O_2$ given in table 9.2, the value of the Faraday given in table 13.1 is obtained. The Marinenko and Taylor benzoic acid measurements were carried out during February and March, 1963, with a mean date of 4 March 1963. We thus convert to BIPM units by linearly interpolating between the 1961 and 1964 BIPM intercomparisons using this date. The interpolation result is $A_{NBS} - A_{BIPM} = (-1.82 \pm 0.16)\ \mu A$. The uncertainty assigned the benzoic acid Faraday is the RSS of the 1.4 ppm uncertainty in the molecular weight of benzoic acid (table 9.2), the 5.2 ppm statistical standard deviation of the mean of the 19 coulometric measurements, and the following systematic uncertainties which have been estimated from the paper of Marinenko and Taylor [13.2, 13.3]: 2 ppm for weighing and the standard masses used; 1 ppm each for the voltage reference, the resistance standard, and the time standard; and 10 ppm for the effect of impurities.

For oxalic acid dihydrate, Marinenko and Taylor find $E(C_2H_2O_4 \cdot 2H_2O) = 0.6532925 \times 10^{-6}$ kg/A_{NBS}·s. Using the value of the molecular weight for $C_2H_2O_4 \cdot$

TABLE 13.1. Summary of Faraday measurements

Publication date, laboratory, and author	Material	F_{LAB} ($A_{LAB} \cdot s \cdot mol^{-1}$)	F_{BIPM} ($A_{BIPM} \cdot s \cdot mol^{-1}$)	F_{BI69} ($A_{BI69} \cdot s \cdot mol^{-1}$)	Uncertainty (ppm)	Eq. No.
1960, NBS Craig et al.[a]	Silver	96485.70(66)	96485.66(66)	96486.72(66)	6.8	(13.1)
1968, NBS Marinenko and Taylor[b]	Benzoic Acid	96486.42(1.12)	96486.24(1.12)	96487.30(1.12)	12	
	Oxalic Acid	96485.37(1.57)	96485.19(1.57)	96486.25(1.57)	16	
	Average of benzoic and oxalic acid values[d]			96486.95(93)	9.6	(13.2)
1971, NBS Bower[c]	Iodine	96485.44(1.48)		96485.36(1.48)	15[e]	(13.3)

[a] Refs. [0.1, 13.1]. [b] Refs. [13.2, 13.3]. [c] Refs. [13.4, 13.5]. [d] See text.
[e] Statistical uncertainty only.

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$2H_2O$ given in table 9.2 yields the value of F given in table 13.1. (Note that this material is divalent.) The benzoic acid measurements were carried out during July, 1963, with a mean date of 22 July 1963. Using this date to interpolate between the 1961 and 1964 intercomparisons yields $A_{NBS} - A_{BIPM} = (-1.87 \pm 0.16)\ \mu A$, which we use to convert to BIPM units. The uncertainty assigned F is the RSS of the 2.0 ppm uncertainty in the molecular weight of oxalic acid dihydrate (table 9.2), the 5.4 ppm statistical standard deviation of the mean of the 11 coulometric measurements, and the same systematic uncertainties as for the benzoic acid measurements with the exception that the impurity uncertainty is estimated to be 15 ppm.

Since the benzoic and oxalic acid measurements were carried out under very similar conditions and using similar methods, we choose to combine them to obtain a single value for possible use in our adjustment. The final result is given in the table and has been obtained by first subtracting out from each value the common systematic uncertainties, taking a weighted mean, and then adding back the systematic uncertainties.

(c) NBS iodine measurement. Bower's result [13.4] is very preliminary and is included for completeness only; it is based on but four runs. (The value quoted differs from that given in ref. [13.4] because of a new analysis of the data [13.5].) Because this is a post 1 January 1969 measurement, we convert to BI69 units using eqs (2.5b) and (2.6b), and eqs (4.5a) and (4.5b). The iodine runs were made from January to March, 1971, with a mean date of 3 April 1971. This date yields $V_{NBS} - V_{BI69} = (-0.76 \pm 0.04)\ \mu V$, $\Omega_{NBS} - \Omega_{BI69} = (0.06 \pm 0.08)\ \mu\Omega$, and thus $A_{NBS} - A_{BI69} = (-0.82 \pm 0.09)\ \mu A$. The uncertainty assigned the iodine Faraday is solely statistical since no attempt has yet been made to estimate the systematic uncertainties. We note that all four Faraday values in table 13.1 are in surprisingly good agreement.

14. Proton Gyromagnetic Ratio, γ'_p

The proton gyromagnetic ratio is now of critical importance in any least-squares adjustment since a precise value of the fine-structure constant may be obtained from low field measurements of γ'_p, and $2e/h$ from the ac Josephson effect [0.1]. Table 14.1 summarizes the measurements of interest. (In several instances, we have leaned heavily on the analysis of Taylor et al. [0.1].)

TABLE 14.1. Summary of γ'_p determinations

Publication date, laboratory[a], and author	γ'_p	γ'_p	γ'_p	Uncertainty (ppm)	Eq. No.
		Low Field			
	$10^8\ s^{-1}\cdot T^{-1}_{LAB}$	$10^8\ s^{-1}\cdot T^{-1}_{BIPM}$	$10^8\ s^{-1}\cdot T^{-1}_{BI69}$		
1968, ETL Hara et al.[b]	2.6751384(107)	2.6751449(107)	2.6751156(107)	4.0	(14.1)
1972, NBS Olsen and Driscoll[c]	2.6751344(54)		2.6751370(54)	2.0	(14.2)
1965, NPL Vigoureux[d]	2.6751707(107)	2.651480(107)	2.6751187(107)	4.0	(14.3)
1971, VNIIM Malyarevskaya, Studentsov, and Shifrin[e]	See text.		2.6751100(161)	6.0	(14.4)
		High Field			
	$10^8\ A_{LAB}\cdot s\cdot kg^{-1}$	$10^8\ A_{BIPM}\cdot s\cdot kg^{-1}$	$10^8\ A_{BI69}\cdot s\cdot kg^{-1}$		
1966, KhGNIIM Yagola, Zingerman, and Sepetyi[f]	2.675079(20)[h]	2.675101(20)	2.675130(20)	7.4	(14.5)
1971, NPL Kibble and Hunt[g]	2.675075(43)		2.675075(43)	16	(14.6)

[a] ETL = Electrotechnical Laboratory, Japan; KhGNIIM = Kharkov State Scientific Research Institute of Metrology, U.S.S.R.
[b] Refs. [0.1, 14.2]. [c] Ref. [14.3]. [d] Refs. [0.1, 14.4]. [e] Refs. [14.5, 14.6]. [f] Refs. [0.1, 14.7, 14.8].
[g] Refs. [14.9, 14.10]. [h] This result is in terms of A_{MM}, the ampere as maintained at VNIIM.

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It should be noted that the high field and low field measurements of γ_p' in practice determine two different quantities because of the different manner in which the as-maintained ampere enters the experiment. For the low field measurements the field is expressed directly in terms of $A_i \cdot m^{-1}$, but for the high field measurements the field is in terms of $N_i \cdot A_i^{-1} \cdot m^{-1}$, where A_i and N_i are the ampere and the newton as maintained in the local laboratory [14.1]. The local realization of the newton is inversely proportional to g_i, the local acceleration of gravity. In our present adjustment, γ_p'(low) is then independent of K, the ampere conversion factor, while γ_p'(high) is proportional to K^{-2} (see the observational equations for these quantities in table 29.1). Since $\gamma_p' = (\mu_p'/\mu_N)e/m_p$, it is convenient to use the suggestion of Huntoon and McNish [14.1] and express γ_p'(high) in units of $A \cdot s \cdot kg^{-1}$. We also note that high and low field measurements in the same laboratory constitute a direct determination of the absolute ampere in which the proton resonance frequency serves only to transfer a low field measurement (in which the field is calculated from the magnetic coil geometry) to a high field measurement (in which the field is measured in terms of mechanical forces).

We make the following specific comments with regard to the data of table 14.1.

(a) ETL. Additional information provided to us by the experimenters [14.11] has clarified several of the questions concerning this work which were raised in ref. [0.1]. We now believe it may be considered for inclusion in an adjustment. The value in the table is the 1968 result as reported by Hara et al. and quoted in ref. [0.1]. The 4 ppm assigned uncertainty is that recommended by the experimenters [14.11] and is only slightly larger than the actual RSS of their estimated uncertainties as originally given [14.2, 14.12]. We also believe it to be realistic relative to the 2.0 ppm uncertainty assigned the NBS result (to be discussed next). The latter experiment is probably the most complete carried out to date.

We have converted to BIPM units using the relation $A_{ETL} - A_{BIPM} = (-2.44 \pm 0.16)\ \mu A$ which was obtained by linearly interpolating between the 1967 and 1970 triennial intercomparisons, taking into account the 1 January 1969 redefinitions in V_{ETL} and V_{BIPM}. (The ETL measurements were carried out during April 1968 with a mean date of 7 April.)

(b) NBS. The value given is the August, 1971 result reported by Olsen and Driscoll in ref. [14.3], and includes the 0.3 ppm "bending" correction indicated in the "Note Added in Proof" of that paper. It is believed to be by far the most reliable of all of the NBS determinations since the pitch of the precision solenoid used in the experiment was measured using a laser interferometer; and numerous corrections were carefully considered. (These pitch measurements have been more or less confirmed by the preliminary work of Williams and Olsen [14.13] who are using a magnetic pickup probe to detect wire position rather than the contacting probe used by Olsen and Driscoll.) In view of the superior nature of the new NBS measurements, we shall neglect those carried out prior to it (see ref. [0.1] for a detailed summary). It should be noted however, that the mean of all the pre 1971 measurements is only 1.6 ppm less than the 1971 result, well within the 3.7 ppm uncertainty of the former. Furthermore, the difference may be attributed in part to the inclusion of additional corrections in the new work which were omitted in the older work. The 2 ppm uncertainty assigned the new result by Olsen and Driscoll and which we have adopted would appear to be conservative since they obtained it by doubling their original one ppm estimate in order to allow for the somewhat limited nature of their measurements.

Since this is a post 1 January 1969 determination, we convert to BI69 units using eqs (2.5b), (2.6b), (4.5a), and (4.5b). Taking 19 August 1971 as the mean date of the experiment, we find $V_{NBS} - V_{BI69} = (-0.88 \pm 0.03)\ \mu V$, $\Omega_{NBS} - \Omega_{BI69} = (0.07 \pm 0.08)\ \mu\Omega$, and thus $A_{NBS} - A_{BI69} = (-0.96 \pm 0.09)\ \mu A$.

(c) NPL (low field). The value given is taken from ref. [0.1]. We convert to BIPM units using $A_{NPL} - A_{BIPM} = (8.5 \pm 0.16)\ \mu A$ as obtained from the 1961 BIPM intercomparison (6 January central date), since the NPL γ_p' measurements were carried out during January and February of 1961. The assigned uncertainty is our own estimate and has been taken to be equal to that assigned the ETL result since the effort put into each was comparable. This uncertainty assignment is also more in keeping with the 2.6 ppm uncertainty [0.1] implied by Vigoureux's [14.4] own original estimates (interpreted as limits of error) than the 5.8 ppm assigned in ref. [0.1]. It is also realistic relative to the uncertainty assigned the NBS result.

(d) VNIIM. The low field γ_p' measurements carried out over the period 1958 to 1968 at the Mendeleev All-Union Scientific Research Institute of Metrology, U.S.S.R., by Studentsov, Yanovskii, and others [14.5, 14.6, 14.14, 14.15] were discussed in some detail in ref. [0.1]. However, the VNIIM result was not included in the 1969 adjustment of Taylor et al. because they did not believe they had sufficient information to properly assess the uncertainty to be assigned the VNIIM work. We have now obtained additional data [14.5, 14.16] concerning these experiments and have derived a value of γ_p' for possible inclusion in the present adjustment.

The data we consider are summarized in table 14.2. The first column gives the number of the Helmholtz coil used in a particular measurement, the second column gives the year the measurement was carried out,[11] and the third column gives the result. (All of these data are taken from ref. [14.5].) We convert to BIPM units by linear interpolation between BIPM

[11] It should be borne in mind, however, that the dimensions of the helmholtz rings were remeasured with improved techniques between 1966 and 1968, and the revised values were used to correct the earlier γ_p' results [0.1, 14.5].

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TABLE 14.2. Summary of VNIIM low field γ'_p measurements[a]

Coil No.	Year	γ'_p ($10^8\ s^{-1}\cdot T_{IMM}^{-1}$)	γ'_p ($10^8\ s^{-1}\cdot T_{BIPM}^{-1}$)	γ'_p ($10^8 s^{-1}\cdot T_{BI69}^{-1}$)	Mean ($10^8 s^{-1}\cdot T_{BI69}^{-1}$)	Standard deviation (ppm)
4	1960	2.675167	2.6751467	2.6751174	2.6751174	
5	1960	2.675131	2.6751107	2.6750814		
	1961	156	1354	1061		
	1962	168	1464	1170	2.6751015(105)	3.9
6	1960	2.675148	2.6751277	2.6750984		
	1963	175	1523	1230	2.6751107(123)	4.6
7	1960	2.675160	2.6751397	2.6751104		
	1961	177	1564	1271		
	1962	135	1134	0840		
	1966	153	1293	1000		
	1967	164	1408	1114	2.6751066(71)	2.7
8	1962	2.675125	2.6751034	2.6750740		
	1967	150	1268	0974	2.6750857(117)	4.4
9	1960	2.675147	2.6751267	2.6750974		
	1961	161	1404	1111		
	1962	183	1614	1320		
	1966	182	1583	1290		
	1967	136	1128	0834	2.6751106(92)	3.5
10	1961	2.675136	2.6751154	2.6750861		
	1962	128	1064	0770		
	1966	149	1253	0960		
	1967	169	1458	1164	2.6750939(85)	3.2
11	1962	2.675153	2.6751314	2.6751020		
	1966	147	1233	0940		
	1967	191	1678	1384	2.6751115(137)	5.1
12	1968	2.675156	2.6751343	2.6751050	2.6751050	
13	1962	2.675179	2.6751574	2.6751280		
	1966	173	1493	1200		
	1967	188	1648	1354	2.6751278(45)	1.7
14	1962	2.675197	2.6751754	2.6751460		
	1966	181	1573	1280		
	1967	239	2158	1864		
	1968	183	1613	1320	2.6751481(133)[b]	5.0
15	1962	2.675179	2.6751574	2.6751280		
	1963	141	1183	0890		
	1965	163	1395	1101		
	1966	166	1423	1130		
	1967	179	1558	1264	2.6751133(70)	2.6

[a] Ref. [14.5]. [b] Deleting the 1967 value gives 2.6751353(55) (2.0 ppm); see text.

triennial comparisons taking 1 July of the year in question as the mean date of each measurement. The relevant ampere differences are:

$$1960: A_{IMM} - A_{BIPM} = 7.6\ \mu A,$$

$$1961: A_{IMM} - A_{BIPM} = 7.7\ \mu A,$$

$$1962: A_{IMM} - A_{BIPM} = 8.1\ \mu A,$$

$$1963: A_{IMM} - A_{BIPM} = 8.5\ \mu A,$$

$$1965: A_{IMM} - A_{BIPM} = 8.8\ \mu A,$$

$$1966: A_{IMM} - A_{BIPM} = 8.9\ \mu A,$$

$$1967: A_{IMM} - A_{BIPM} = 8.7\ \mu A,$$

$$1968: A_{IMM} - A_{BIPM} = 8.1\ \mu A.$$

(It should be emphasized that converting to BIPM units does not introduce any additional scatter into the

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data, but rather decreases it somewhat.) The last two columns of table 14.2 give the mean and standard deviation of the mean (statistical only) of the measurements made with each coil.

The mean within-coil variance, $\hat{\sigma}_W^2$, may be calculated from [14.11]:

$$\hat{\sigma}_W^2 = \left[\sum_{j=4}^{15} \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \right] \Big/ \sum_{j=4}^{15} (n_j - 1), \quad (14.7a)$$

which gives

$$\hat{\sigma}_W = 0.0000188 \times 10^8 \text{ s}^{-1} \cdot \text{T}_{\text{Bi69}}^{-1} \text{ (7.0 ppm)}. \quad (14.7b)$$

Here, n_j is the number of measurements carried out with the jth coil, x_{ij} is the ith measurement carried out with the jth coil, $\bar{x}_j$ is the mean value obtained with each coil.:

$$\bar{x}_j = \left(\sum_{i=1}^{n_j} x_{ij} \right) \Big/ n_j, \quad (14.7c)$$

$N = 38 = \sum_{j=4}^{15} n_j$ is the total number of measurements, and $\sum_{j=4}^{15} 1 = J = 12 =$ the number of coils.

The between-coil variance, $\hat{\sigma}_B^2$, is given by [14.11]

$$\hat{\sigma}_B^2 = \frac{N}{N^2 - \sum_{j=4}^{15} n_j^2} \cdot \left(\sum_{j=4}^{15} n_j(\bar{x}_j - \bar{x})^2 - (J - 1)\hat{\sigma}_W^2 \right); \quad (14.8a)$$

$$\hat{\sigma}_B = 0.0000125 \times 10^8 \text{ s}^{-1} \cdot \text{T}_{\text{Bi69}}^{-1} \text{ (4.7 ppm)}, \quad (14.8b)$$

where, $\bar{x} = \left(\sum_{j=4}^{15} n_j \bar{x}_j \right) \Big/ N$, that is, $\bar{x}$ is the mean of all of the measurements. An F test of the statistical significance of this value of $\hat{\sigma}_B^2$ may be obtained from the ratio [14.17]

$$F_{11/26} = \left(\frac{\sum_{j=4}^{15} n_j(\bar{x}_j - \bar{x})^2}{J - 1} \right) \Big/ \hat{\sigma}_W^2, \quad (14.9a)$$

$$= 2.37. \quad (14.9b)$$

This F value for 11 and 26 degrees of freedom is statistically significant at the 95 percent confidence level [14.18]. However, we note that the 1967 measurement on coil 14 contributes disproportionately to both $\hat{\sigma}_W^2$ and $\hat{\sigma}_B^2$ and is clearly discrepant. (It differs from the mean of all 38 measurements by 3.33 times the standard deviation of these measurements.) Deleting this result, which changes N and n_{14} to 37 and 3, respectively, yields

$$\hat{\sigma}_W = 0.0000170 \times 10^8 \text{ s}^{-1} \cdot \text{T}_{\text{Bi69}}^{-1} \text{ (6.4 ppm)}, \quad (14.10a)$$

$$\hat{\sigma}_B = 0.0000083 \times 10^8 \text{ s}^{-1} \cdot \text{T}_{\text{Bi69}}^{-1} \text{ (3.1 ppm)}, \quad (14.10b)$$

$$F_{11/25} = 1.72. \quad (14.10c)$$

Since this value of F is no longer statistically significant (the critical or 95 percent confidence level F value for 11 and 25 degrees of freedom is 2.20 [14.18]), we may conclude that there is no significant between-coil component of variance. Thus, the average of the 37 separate measurements is to be preferred over the unweighted mean of the 12 mean values obtained with each coil. The result is

$$\gamma_p' = 2.6751100(31) \times 10^8 \text{ s}^{-1} \cdot \text{T}_{\text{Bi69}}^{-1} \text{ (1.2 ppm)}. \quad (14.11)$$

(For comparison purposes, we note that the mean of the 12 values is 2.6751099(39) (1.5 ppm); for all 38 measurements, the respective numbers are 2.6751120(36) (1.4 ppm) and 2.6751110(46) (1.7 ppm).)

We take eq (14.11) as the final result of the VNIIM low field work but with an uncertainty of 6.0 ppm which is based on the above 1.2 ppm statistical uncertainty; a 3 ppm systematic uncertainty as assigned by Malyarevskaya et al. [14.5] due to a variety of sources (the pitch and diametral measurements, phase distortions of the amplification system used to detect the proton precession signal, uncertainties in the electrical standards used and in various temperature corrections); and an additional 5 ppm due to other sources which were originally considered as random [14.5, 14.16] and therefore averaged out, but which we believe must be considered at least partly systematic. That is, we are reluctant to assume that the systematic errors in this experiment have been completely randomized by the multi-coil and multi-year procedure. These additional systematics include such things as pitch and diameter variations, influence of the conductors carrying current to the coils, proton sample alignment, phase distortions of the proton precession signal, as well as a possible time dependence of the γ_p' measurements. (A least-squares fitted straight line to the data of table 14.2 plotted as a function of time (37 points) shows γ_p' to be increasing at a rate of (5.8 ± 4.2) ppm per decade.)

We note that with the exception of the NBS result, all of the low field γ_p' values are in good agreement; a detailed analysis will be given in part III.

(e) KhGNIIM. The Kharkov result is taken directly from Taylor et al. [0.1] but a + 0.2 ppm correction has been applied in order to account for the new IGSN71 value of the acceleration due to gravity at Kharkov, eq (5.1c). We convert to BIPM units using $A_{\text{IMM}} - A_{\text{BIPM}}$

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= (8.1 ± 0.16) μA, which follows from the 1961 and 1964 BIPM triennial intercomparison results and the assumptions made in ref. [0.1] regarding the time period of the measurements.

(f) NPL (high field). Using a variable width Cotton balance coil, Kibble and Hunt completed a preliminary γ'_p high field measurement in 1970 using a prototype apparatus. (The work is continuing with a significantly improved apparatus [14.19]). The result given in the table is as quoted by Kibble and Hunt except that a +0.07 ppm correction has been applied due to the new IGSN71 value of g(BFS), eq (5.1b). [These workers used 9.8118132 m/s^2 as the value of g at the site of their balance, based on the mean of the Cook [5.2] and Faller and Hammond [0.1] absolute measurements of g(BFS).] The final uncertainty assigned by Kibble and Hunt is the RSS of an 11.7 ppm random component and an 11.6 ppm estimated systematic component.

Since this is a post 1 January 1969 experiment, we convert to A_{B169} using eqs (2.5c), (2.6c), (4.5c), and (4.5d). The measurements were carried out from April to July 1970 with a mean date of 16 June. This yields $V_{NPL} - V_{B169} = (0.28 \pm 0.14)$ μV, $\Omega_{NPL} - \Omega_{B169} = (0.30 \pm 0.04)$ $\mu\Omega$, and thus $A_{NPL} - A_{B169} = (-0.01 \pm 0.15)$ μA. We note that the two high field determinations differ by less than 1.2 combined standard deviations (RSS). Although their uncertainties are large compared with those assigned the low field values, they are of importance because the values of the ampere conversion factor, $K \equiv A_{B169}/A$, which follow from the relation [0.1]

$$K = [\gamma'_p(\text{low})_{B169}/\gamma'_p(\text{high})_{B169}]^{1/2}, \qquad (14.12)$$

have uncertainties comparable with those assigned the direct values of K obtained from the Pellat and current balance experiments, table 12.1. (We defer a discussion of the overall agreement between the various values of γ'_p(low), γ'_p(high), and K to part III.)

15. Magnetic Moment of the Proton in units of the Nuclear Magneton, μ'_p/μ_N

Deciding how to handle the apparent inconsistencies among the available μ'_p/μ_N measurements was the major problem facing Taylor et al. in their 1969 adjustment. However, this difficulty has now been removed with the advent of the reanalysis (using the original data notebooks) of the Sommer, Thomas, and Hipple μ'_p/μ_N measurement by Fystrom, Petley and Taylor; and the recent high precision sub-ppm measurements of μ'_p/μ_N by Mamyrin, Aruyev, and Alekseenko using their unique resonance mass spectrometer, and by Petley and Morris using an omegatron. For both groups, the sub-ppm results represent the culmination of research programs extending over a number of years. Table 15.1 summarizes the various μ'_p/μ_N determinations and gives their associated references. The following comments apply.

(a) Sommer et al. The reanalysis of the Sommer et al. [15.1] experiment by Fystrom et al. [15.2] led to a +9.2 ppm correction to Sommer et al.'s originally reported result and to the conclusion that their assigned uncertainty should be interpreted as corresponding to one standard deviation rather than "several times the probable error" as Sommer et al. stated in their original publication [15.1].

(b) Mamyrin et al. Mamyrin and coworkers considerably improved their magnetic resonance spectrometer, primarily by introducing a compound ion source that produces two ion species at the same time. This permits them to correct for the effect of stray electric fields to very high accuracy. The value of μ'_p/μ_N given by Mamyrin et al. [15.9], 2.7927744(12) (0.43 ppm), was obtained using the 1965 atomic mass values [15.16]. The result given in table 15.1 is our own revision of their result taking into account the new atomic masses of Wapstra, Gove, and Bos, table 9.1.

(c) Petley and Morris. These workers have made significant advances in the design of their omegatron and in improving their entire experiment. These include working in a higher and more uniform magnetic field, and varying the applied r.f. at fixed magnetic field. The latter has reduced the statistical standard deviation of a single measurement by a factor of ten compared with scanning the magnetic field as in their earlier work. The result quoted in table 15.1 is based on the atomic masses given in table 9.1.

The good agreement between the μ'_p/μ_N measurements summarized in table 15.1, especially between the two high precision determinations, eqs (15.7) and (15.8), and the three medium precision determinations, eqs (15.4–15.6), gives added assurance of the absence of systematic errors since widely differing measurement techniques were used. However, in spite of this overall good agreement, we choose to include in our adjustment only the high precision values of Mamyrin et al. and Petley and Morris, eqs (15.7) and (15.8). This decision follows from the fact that in general, it is not good practice to include in an adjustment values of the same quantity which have uncertainties which differ by more than about a factor of three; see ref. [0.1] for a discussion of this point.

16. Ratio, kxu to ångström, Å

In the present work, we shall follow Taylor et al. [0.1] and express the results of various x-ray experiments on the x-unit scale defined by taking the peak intensity of the Cu$K\alpha_1$ line to be

$$\lambda(\text{Cu}K\alpha_1) \equiv 1.5374000 \text{ kxu}. \qquad (16.1)$$

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TABLE 15.1. Summary of measurements of μ_p'/μ_N

Publication date and author	Method	Value	Uncertainty (ppm)	Eq. No.
1951, Sommer, Thomas, and Hipple[a] (Revised 1971, Fystrom, Petley, and Taylor[b])	Omegatron	2.792711(60)	21	(15.1)
1963, Sanders and Turberfield[c]	Inverse cyclotron	2.792701(73)	26	(15.2)
1961, Boyne and Franken[d]	Cyclotron	2.792832(55)	20	(15.3)
1970, Fystrom[e]	Omegatron	2.792783(16)	5.7	(15.4)
1972, Luxon and Rich[f]	Trapped ion	2.792786(17)	6.0	(15.5)
1972, Staub[g]	Velocity gauge	2.792777(20)	7.2	(15.6)
1972, Mamyrin, Aruyev, and Alekseenko[h]	Mass spectrometer	2.7927738(12)	0.43	(15.7)
1972, Petley and Morris[i]	Omegatron	2.7927748(23)	0.82	(15.8)

[a] Ref. [15.1]. [b] Ref. [15.2]. [c] Refs. [0.1, 15.3]. [d] Refs. [0.1, 15.4]. [e] Refs. [15.5, 15.6]. [f] Ref. [15.7].
[g] H. Staub, private communication, and to be published. This is the most recent result of the Zurich group. See ref. [15.8] for their earlier work.
[h] Ref. [15.9] and text. This is the most recent result of Mamyrin's group; see refs. [15.10–15.12] for their earlier work.
[i] B. W. Petley, private communication, and ref. [15.13]. This is Petley and Morris' most recent result; see refs. [15.14, 15.15] for their earlier work.

TABLE 16.1. Summary of values of Λ

Publication date and author	Method[a]	Value[b]	Uncertainty (ppm)	Eq. No.
1931, Bearden[c] (Revised 1964, I. Henins and Bearden[d])	PRG	1.002027(33)	33	(16.3)
1940, Tyrén[e] (Revised 1965, Edlén and Svensson[f]; reanalysed 1969, Noreland et al.[g])	CRG	1.002027(33)	33	(16.4)
1971, A. Henins[h]	PRG	1.0020655(98)	9.8	(16.5)
1972, Deslattes and Sauder[i]	XROI	1.0020841(24)	2.4	(16.6)
1964, Spijkerman and Bearden[j]	SWL	1.002041(33)	33	(16.7)

[a] PRG = plane ruled grating. CRG = concave ruled grating. XROI = x-ray-optical interferometer. SWL = short wavelength limit.
[b] x-unit scale defined by $\lambda(\mathrm{Cu}K\alpha_1) \equiv 1.537400$ kxu.
[c] Ref. [16.3]. [d] Ref. [16.4]. [e] Ref. [16.5]. [f] Refs. [16.6, 16.7].
[g] Ref. [16.8]. [h] Ref. [16.9]. [i] Ref. [16.10]. [j] Ref. [16.11].

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On this same scale, Bearden and coworkers find the following secondary standards [16.1, 16.2]:

$$\lambda(\mathrm{W}K\alpha_1) = 0.2085810(4)\ \mathrm{kxu}\ (1.8\ \mathrm{ppm}), \tag{16.2a}$$

$$\lambda(\mathrm{Ag}K\alpha_1) = 0.5582594(9)\ \mathrm{kxu}\ (1.6\ \mathrm{ppm}), \tag{16.2b}$$

$$\lambda(\mathrm{Mo}K\alpha_1) = 0.7078448(10)\ \mathrm{kxu}\ (1.4\ \mathrm{ppm}), \tag{16.2c}$$

$$\lambda(\mathrm{Cr}K\alpha_1) = 2.2888988(38)\ \mathrm{kxu}\ (1.6\ \mathrm{ppm}). \tag{16.2d}$$

The quoted uncertainties are standard deviations.[12] These wavelengths are not statistically independent; one may use a correlation coefficient of 0.4 between any pair.

The kx-unit is slightly larger than 10^{-10} m. The ratio, Λ, relating the kx-unit to the metre is defined by the relation

$$\Lambda \equiv \lambda(10^{-10}\mathrm{m})/\lambda(\mathrm{kxu}). \tag{16.2}$$

That is, Λ is defined as the ratio of a wavelength expressed in 10^{-10} m to the same wavelength expressed in kilo-x-units, kxu. (Recall 1 ångström $\equiv$ 1Å $\equiv 10^{-10}$ m.) Table 16.1 summarizes the relevant measurements of Λ.

(a) Bearden. Bearden's data [16.3] have been reviewed by DuMond [16.12] and by I. Henins and Bearden [16.4], and although some forty years old, the results still appear valid when corrected to the wavelength scale characterized by eqs (16.1) and (16.2). The value given in the table is the simple mean of the $\mathrm{Cu}K\beta_{1,3}$, $\mathrm{Cu}K\alpha_{1,2}$, $\mathrm{Cr}K\beta_{1,3}$ and $\mathrm{Cu}K\alpha_{1,2}$ results given in ref. [16.4] (table VI). The uncertainty quoted is simply 1/3 the 100 ppm limiting error originally assigned the measurements [16.4]. While the standard deviation of the mean of the four values is only 21 ppm, we use the 33 ppm figure in order to account for possible systematic errors in a rather difficult experiment.

(b) Tyrén. Tyrén's [16.5] 5–m concave grating spectrograph measurements of the $\mathrm{Al}K\alpha$ lines as corrected and revised by Edlén and Svensson [16.6, 16.7] were discussed by Taylor et al. [0.1]. Since the several criticisms of this work given there are still valid (arising mainly from the effects of Al_2O_3 on the shape and position of the $\mathrm{Al}K\alpha$ lines), we shall not consider it for possible inclusion in our adjustment. However, for completeness and purposes of comparison we give in the table the value of Λ implied by the Tyrén-Edlen-Svensson data as reanalysed recently by Noreland et al. [16.8]. Their treatment would appear to be as judicious and complete as any one might expect, and includes their own recent measurements of the $\mathrm{Al}K\alpha_{1,2}$ lines. Fortuitously, the result is identical to that of Bearden. (Note that we have converted Noreland et al.'s original Λ value to our adopted $\mathrm{Cu}K\alpha_1$ x-unit scale and their original assigned probable error to a standard deviation.)

(c) A. Henins. Henins [16.9] at Johns Hopkins determined the ratio

$$\lambda(\mathrm{Al}K\alpha_{1,2})/\lambda(\mathrm{Cu}K\alpha_1) = 5.413782(22)\ (4.2\ \mathrm{ppm}), \tag{16.8}$$

using a plane quartz crystal spectrometer. With the same x-ray tube and collimator but with the crystal replaced by a plane ruled grating, he also measured the absolute wavelength of $\mathrm{Al}K\alpha_{1,2}$ with the result

$$\lambda(\mathrm{Al}K\alpha_{1,2}) = 8.34034(7) \times 10^{-10}\ \mathrm{m}\ (8.9\ \mathrm{ppm}). \tag{16.9}$$

These data imply, using the relationship

$$\Lambda = \frac{\lambda(\mathrm{Al}K\alpha_{1,2})}{[\lambda(\mathrm{Al}K\alpha_{1,2})/\lambda(\mathrm{Cu}K\alpha_1)]\cdot 1.5374000\ \mathrm{kxu}}, \tag{16.10}$$

the value

$$\Lambda = 1.0020655(98)\ (9.8\ \mathrm{ppm}). \tag{16.4}$$

Note that using the data in this way eliminates any systematic effects arising from shifts and distortions in the $\mathrm{Al}K\alpha_{1,2}$ lines. In essence, this line is being used as a transfer standard and need only remain constant during the course of the experiments.

(d) Deslattes and Sauder. These workers [16.10] have recently reported the preliminary results of their measurement (in metres) of the repeat distance of the 220 planes of a single crystal of modified-float-zone produced silicon using a combined x-ray-optical interferometer. They find $d_{220} = 1.920170(4) \times 10^{-10}$ m (2 ppm). When a sample of Si taken adjacent to this crystal (which may now be viewed as a calibrated lattice-spacing standard) is used to measure diffraction angles in various orders of (111) for $\mathrm{Mo}K\alpha_1$ radiation, they find $\lambda(\mathrm{Mo}K\alpha_1) = 0.709320 \times 10^{-10}$ m (2 ppm). When combined with the value of $\lambda(\mathrm{Mo}K\alpha_1)$ given in eq (16.2c), the result is $\Lambda = 1.0020841(24)$ (2.4 ppm). However, although it is assigned the lowest uncertainty of the five values given, we shall not consider it for inclusion in our adjustment because of the highly preliminary nature of the work; several possible sources of systematic error have yet to be investigated [16.13].

(e) Spijkerman and Bearden. A value of Λ may be obtained from Spijkerman and Bearden's [16.11] measurement of the short wavelength limit of the contin

[12] All of the uncertainties in the experiments carried out at the Johns Hopkins x-ray laboratory (under Prof. J. A. Bearden) were originally reported in the form of probable errors (P.E.).

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ous x-ray spectrum generated by a specially constructed x-ray tube. Using a Hg vapor jet as the target in order to eliminate band structure effects associated with solid targets, they obtained [0.1, 16.11]:

$$hc/e\Lambda = 12373.15 \pm 0.41\ V_{NBS}\cdot\text{kxu (33 ppm)}, \tag{16.11}$$

as measured on our Cu$K\alpha_1$ scale. To convert to BI69 units, we (1) use the relation $V_{NBS} = V_{BIPM} - (1.99 \pm 0.14)\ \mu V$ as obtained by linearly interpolating between the 1961 and 1964 BIPM triennial intercomparisons (the standard cells used in the experiment were calibrated at NBS during December, 1961); and (2) follow the procedures outlined in sections II. A.2 and 4. The net correction to eq (16.11) is (8.98 ± 0.23) ppm and the result is

$$hc/e\Lambda = 12373.26 \pm 0.41\ V_{BI69}\cdot\text{kxu (33 ppm)}. \tag{16.12}$$

Using our adopted value for $2e/h$ and c, table 11.1, yields $hc/e = 12398.518 \times 10^{-10}\ V_{BI69}\cdot m$, and thus, from eq (16.12), one obtains

$$\Lambda = 1.002041(33)\ \text{(33 ppm)}. \tag{16.7}$$

17. Avogadro Constant from X-Rays, $N_A\Lambda^3$

At present, the limiting factor in the determination of the quantity $N_A\Lambda^3$ from x-ray crystal density measurements is the effective molecular weight (or chemical and isotopic composition) of the crystal used. Because the detailed impurity composition (including whether the impurity was interstitial or substitutional) was not carefully evaluated in earlier experiments (pre 1960), we follow ref. [0.1] and consider for possible inclusion in our adjustment only two determinations: That of I. Henins and Bearden [16.4] using silicon; and that of Bearden [17.1] using calcite ($CaCO_3$). The silicon work gives [0.1, 16.4]

$$N_A\Lambda^3 = 6.059768(95) \times 10^{23}\ \text{mol}^{-1}\ \text{(16 ppm)}, \tag{17.1}$$

TABLE 17.1. Summary of measurements of $N_A\Lambda^3$

Publication date and author	Substance	Value[a] (10^{23} mol^{-1})	Uncertainty (ppm)	Eq. No.
1964, I. Henins and Bearden[b]	Si	6.059768(95)	16	(17.1)
1965, Bearden[c]	Calcite	6.05961(17)	28	(17.2)
1971, Deslattes and Sauder[d]	Si	6.059906(94)	16	(17.4)

[a] x-unit scale defined by $\lambda(\text{Cu}K\alpha_1) \equiv 1.537400$ kxu.
[b] Ref. [0.1, 16.4]. [c] Ref. [17.1]. [d] Ref. [16.10].

based on an atomic weight for Si of 28.0857(4) (15 ppm) [16.4]. Henins and Bearden calculated this weight using the same abundance ratios as given in table 9.1, but with the nuclidic mass values of Everling et al. [17.2]. However, using the nuclidic masses given in table 9.1 yields essentially the same value (see table 9.2) and thus leaves eq (17.1) unchanged.

The original result of the calcite work was given as [17.1, 0.1] $N_A\Lambda^3 = 6.05972(23) \times 10^{23}$ mol^{-1} (37 ppm) based on a molecular weight for calcite of 100.088(3) (30 ppm). The data of table 9.1 give 100.0862(16) (16 ppm) (see table 9.2). Following Bearden [17.1], this yields

$$N_A\Lambda^3 = 6.05961(17) \times 10^{23}\ \text{mol}^{-1}\ \text{(28 ppm)}. \tag{17.2}$$

The silicon and calcite measurements are clearly in good agreement, a satisfying situation in view of the difficult impurity problems associated with the latter [17.1].

For completeness, we mention that a value of $N_A\Lambda^3$ may also be derived from the data presented by Deslattes and Sauder and discussed in the previous section. They report [16.10] that the density of two samples taken adjacent to and on opposite sides of their silicon x-ray crystal interferometer had measured densities of 2.328991 and 2.328995 g/cm^3. If we take the density of the silicon used in the lattice spacing measurements as being equal to the mean with a ± 2 ppm uncertainty, and combine it with the 220 repeat distance result (previous section) and the atomic weight of Si given in table 9.2, we find

$$N_A = 6.022176(97) \times 10^{23}\ \text{mol}^{-1}\ \text{(16 ppm)}. \tag{17.3}$$

Combining this result with eq (16.6) yields[13]

$$N_A\Lambda^3 = 6.059906(94) \times 10^{23}\ \text{mol}^{-1}\ \text{(16 ppm)}. \tag{17.4}$$

We shall not consider either of these results for possible inclusion in our adjustment for the same reasons that the Deslattes-Sauder Λ result was not considered for possible inclusion (see sec.II.B.16). Furthermore, as these authors point out, using the geochemical atomic weight as given in table 9.2 is a questionable assumption in view of the isotopic fractionation which may occur during the float-zone purification of the silicon crystal used in the interferometer experiment.

The three values of $N_A\Lambda^3$ discussed in this section are summarized in table 17.1.

[13] The uncertainty assigned N_A arises from the following components: density, 2 ppm; Si atomic weight, 15 ppm; d^3_{220}, 6 ppm. For $N_A\Lambda^3$ the components are density, 2 ppm; Si atomic weight, 15 ppm; $(d^3_{220})_{kxu}$, 4.1 ppm, arising from the uncertainty in $\lambda(\text{Mo}K\alpha_1)$ in terms of $\lambda(\text{Cu}K\alpha_1)$ [see eq (16.2c)]. Note that the Deslattes-Sauder values of Λ, N_A, and $N_A\Lambda^3$ are all interdependent.

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18. Electron Compton Wavelength, $\lambda_C = h/m_e c$

The three precision measurements presently available of the annihilation radiation of electron-positron pairs, or equivalently, the electron Compton wavelength, $\lambda_C = h/m_e c$, are summarized in table 18.1. The Knowles H_2O result is taken directly from the data given by Taylor et al. [0.1]; Knowles' Ta result follows from the data given by Taylor et al. [0.1][14] and the −1.4 ppm correction suggested by Van Assche et al. [18.3] due to the difference between $\lambda(WK\alpha_1)$ generated using naturally occurring W and $\lambda(WK\alpha_1)$ produced in the decay of ^{182}Ta to ^{182}W. (The magnitude of this isotope shift or correction follows from the work of Chesler and Boehm [18.4].)

Knowles' H_2O result is included in the table for purposes of comparison only. It will not be considered for use as an input datum in our adjustment since Knowles [18.5] has emphasized that it was at best a preliminary experiment: The lattice spacing was never measured for the two calcite crystals used, that is, Bearden's measurement of a few mm^3 of one crystal with a total diffraction volume of 62.5 cm^3 cannot be assumed representative of the average crystal.

Van Assche et al.'s result [18.3] was obtained in a manner quite similar to that used by Knowles in his own Ta work, namely, by comparing the annihilation radiation to the $WK\alpha_1$ line generated in the decay of ^{182}Ta to ^{182}W using a bent crystal diffraction spectrometer. It should be noted that Van Assche et al.'s original data, ref. [18.3], table 1, give λ_C − 24.21269(79) × 10^{-3} kxu (33 ppm) and are based on an x-ray scale where $\lambda(WK\alpha_1) = 0.2085770$ kxu.[15] This value of λ_C becomes that given in the table when converted to the $CuK\alpha_1$ x-unit scale used in the present work.

TABLE 18.1. Summary of measurements of the electron Compton wavelength, $\lambda_C = h/m_e c$

Publication date and author	Stopping material	Value[a] (10^{-3} kxu)	Uncertainty (ppm)	Eq. No.
1962, Knowles[b]	H_2O	24.21265(91)	38	(18.1)
1964, Knowles[c]	Ta	24.21416(37)	15	(18.2)
1971, Van Assche et al.[d]	Ta	24.21315(80)	33	(18.3)

[a] x-unit scale defined by $\lambda(CuK\alpha_1) \equiv 1.537400$ kxu.
[b] Refs. [0.1, 18.1]. [c] Refs. [0.1, 18.2]. [d] Ref. [18.3].

C. The Less Precise QED Data

In this section we primarily discuss those experimental data which must be analyzed using quantum electrodynamic (QED) theory in order to derive potential input items for an adjustment. In most cases, the resulting quantities are values of the fine-structure constant. In dealing with these QED data, we have not attempted to carry out a comprehensive comparison between QED theory and experiment as was done by Taylor et al. [0.1]. Rather, we have analyzed in detail only those experiments which can possibly yield a result of sufficient reliability to be considered for inclusion in our adjustment. This approach is consistent with the main aim of the present paper: The derivation of a set of best values of the constants. More comprehensive discussions are deferred to a future publication. The reader is also referred to Lautrup et al. [19.1] and Erickson [23.1].

19. Anomalous Magnetic Moment of the Electron and Muon, a_e and a_μ

As intimated in section II.A.6, by far the most accurate experimental value of a_e is the Wesley and Rich [6.1] result as corrected by Granger and Ford [6.2]:

$$a_e = 0.0011596567(35)\ (3.0\ \text{ppm}). \qquad (19.1)$$

The quantum electrodynamic theoretical expression for a_e may be written as [0.1]

$$a_e = A(\alpha/\pi) + B(\alpha/\pi)^2 + C(\alpha/\pi)^3 + \ldots, \qquad (19.2a)$$

where

$$A = 1/2; \qquad B = -0.328478\ldots. \qquad (19.2b)$$

Calculation of the coefficient C has received much attention in recent years. (See ref. [19.1] for a summary.) It may be expressed as the sum of 72 Feynman diagrams, grouped into four sets [6.4]: $C = C_1 + C_2 + C_3 + C_4$. C_1 has been evaluated analytically by Mignaco and Remiddi [19.2] who find

$$C_1 = 0.055429\ldots \qquad (19.3)$$

Of the six pairs of diagrams grouped into C_2, one pair each have been evaluated analytically by Billi et al. [19.3], and by Barbieri et al. [19.4]. The net result is $C_{2a} = -0.181913\ldots$ For the other 8 diagrams there are two numerical calculations. Brodsky and Kinoshita [19.5] find $C_{2b} = 0.0263(20)$, while Calmet and Perrottet [19.6] give $C_{2b} = 0.0291(22)$. The uncertainties used here are one-half of the sums of the quoted limits of error of the numerical integrations. Taking the weighted mean of these two plus the analytic (exact) value for C_{2a} gives

$$C_2 = -0.1544(15). \qquad (19.4)$$

For C_3 there is only one numerical calculation, that by Aldins et al. [19.7]:

[14] We use the value $\lambda(WK\alpha_1) = 0.2085810(4)$ kxu (1.8 ppm) (see eq (16.2a), sec. II.B.16), instead of $\lambda(WK\alpha_1) = 0.2085811$ as in [0.1].
[15] This x-unit scale is defined by $\lambda(MoK\alpha_1) \equiv 0.707831$ kxu.

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$$C_3 = 0.36(4). \quad (19.5)$$

The fourth set of diagrams was calculated numerically by Levine and Wright [19.8] who found $C_4 = 1.23(20)$; a preliminary result of an improved recalculation by these workers has also been reported [19.9, 19.10]: $C_4 = 0.89(10)$. The most accurate evaluation of C_4 to date is due to Kinoshita and Cvitanovic [6.4] who find

$$C_4 = 1.024(40). \quad (19.6)$$

Summing these independent components [eqs (19.3) – (19.6)] gives

$$C = 1.285 \pm 0.057. \quad (19.7)$$

Combining eqs (19.1), (19.2), and (19.7) finally yields for the fine-structure constant, α,

$$\alpha^{-1}(a_e) = 137.03563(42)\ (3.1\ \text{ppm}). \quad (19.8)$$

Because most of the contributions to C have been obtained by at least two groups and the numerical calculations and exact analytical results (where both exist) are in agreement, serious consideration may now be given to including $\alpha^{-1}(a_e)$ in a least-squares adjustment.

For completeness and future reference we note that the theoretical result for the anomalous moment of the muon, a_μ, may be taken as [0.1, 19.1]:

$$a_\mu = 1/2(\alpha/\pi) + 0.76578(\alpha/\pi)^2 + (22.96 \pm 0.17)(\alpha/\pi)^3 + (68 \pm 9) \times 10^{-9}. \quad (19.9)$$

In calculating $a_\mu^{(6)}$ from the difference $a_\mu^{(6)} - a_e^{(6)}$, where $a^{(6)}$ is the sixth order contribution to the anomalous moment (i.e., the $(\alpha/\pi)^3$ term), we have used the result given in eq (19.6), Peterman's new result for the so called light-by-light scattering contribution [19.11], $a_\mu^{\gamma-\gamma} = 19.76 \pm 0.16$ (which is in good agreement with the value 18.4 ± 1.1 first obtained by Aldins et al. [19.7]); and the results tabulated in ref. [19.1] for the other contributions but with the inclusion of the recent analytic (exact) expressions given in refs. [19.2, 19.3, 19.4] for various terms. The last term in eq (19.9) is the hadronic contribution as calculated by Bramòn, Etim, and Greco [19.12]. We neglect the estimated $(\alpha/\pi)^4$ and weak interaction contributions because they are relatively small and still somewhat speculative [19.13 – 19.16].

Using the CERN experimental result for a_μ given earlier, eq (6.2), and eq (19.9), we find

$$\alpha^{-1}(a_\mu) = 137.0053(363)\ (265\ \text{ppm}). \quad (19.10)$$

This result is consistent with $\alpha^{-1}(a_e)$, eq (19.8), and as we shall see, other values of α.

20. Ground State Hyperfine Splitting in Hydrogen, Muonium, and Positronium: Theory

The equation for the hyperfine splitting in the ground state of a hydrogen-like system can be written in terms of the Fermi-Breit expression corrected for vacuum polarization and other QED manifestations, relativistic corrections, nuclear "recoil", and possible "internal" nuclear structure. For positronium, one must also include additional terms which arise from the virtual annihilation of the electron-positron pair. One may thus write [20.1, 20.2, 20.3, 0.1]

$$E = \frac{16}{3}R_\infty c\alpha^2\left(1+\frac{3}{2}\alpha^2\right)\left(\frac{\mu_e}{\mu_B}\right)\left(\frac{\mu_x}{\mu_B}\right)\left(\frac{m_x}{m_x+m_e}\right)^3 \times\left[1 + A + \frac{\alpha}{\pi}\left(R_1 + \frac{a_x^2}{1+a_x}\frac{m_e}{m_x}R_2\right) + \frac{m_e m_x}{(m_e+m_x)^2} \times\left(\alpha^2\ln\frac{1}{\alpha}\right)R_3 + \alpha^2 Q + \delta_N^{(2)}\right], \quad (20.1)$$

where m_e and m_x are, respectively, the masses of the electron and the nucleus in question (m_p, m_μ or m_e for proton, muon or positron), μ_x is the appropriate nuclear magnetic moment, and $a_x = (g_x/2)-1$ is the anomalous part of the nuclear magnetic moment. The factor $1 + 3\alpha^2/2$ is the Breit relativistic correction to the density of the electron wave function at the nucleus and A is the annihilation term [20.4]:

$$A = \frac{3}{4} - \frac{\alpha}{\pi}\left(\frac{35}{12} + \frac{3}{2}\ \ln 2\right) + \ldots, \quad (20.2)$$

which is to be included only for positronium. We note that the higher order terms in A have not yet been computed. Barbieri et al. [20.5] and Owen [20.6] have calculated a fourth order vacuum polarization contribution to A of $(-\alpha^2/4)\ln\frac{1}{\alpha}$ but other contributions which have not yet been calculated may cancel some or all of this term. Thus A must be considered as known only to an accuracy of the order of 50 ppm.

The nuclear recoil terms R_1 and R_2 have been reviewed by Taylor et al. [0.1] for hydrogen and muonium. The expressions given there can be extended to include positronium if one writes [20.4]

$$R_1 = -\frac{3m_x m_e}{m_x^2 - m_e^2}\left(\frac{2}{g_e}\right)\left(\frac{2}{g_x}\right)\ln\frac{m_x}{m_e}. \quad (20.3)$$

For positronium and muonium a_x^2 is of the order $(\alpha/2\pi)^2$ and hence the R_2 term may be neglected; for hydrogen $R_2 = -16.5 \pm 0.6$. The R_3 recoil term has been recently calculated by Cole and Repko [20.7] who find

$$R_3 = \frac{1}{2(1+a_x)}\left[9+7a_x(1+a_x) - \frac{m_e}{m_x}a_x\,(3+14a_x)\right]. \quad (20.4)$$

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Unfortunately the recoil calculations are incomplete; terms of order $(m_e/m_x)\alpha^2$, (but not containing $\ln \frac{1}{\alpha}$) have as yet only been estimated. Such terms can contribute on the order of 100 ppm, 2 ppm, and 0.5 ppm, respectively, to the hyperfine splitting in positronium, muonium, and hydrogen [20.1].

The quantum electrodynamic terms, Q, have also been reviewed by Taylor et al. [0.1]; for hydrogen and muonium one has

$$Q = \frac{3}{4} - \frac{13}{4} + \ln 2 - \frac{\alpha}{\pi}(57.9 \pm 2). \quad (20.5)$$

(The uncertainty quoted here is a better 1 standard deviation estimate than the value ±5 used by Taylor et al., which was originally intended by Brodsky and Erickson to be a limit of error [20.2, 20.8].) For positronium all but the first term in Q is doubled [20.4].

Based on the theoretical analyses of several authors, Taylor et al. [0.1] were able to limit $\delta_N^{(2)}$, the effect of proton polarizability and internal proton structure, to 0±5 ppm. ($\delta_N^{(2)}$ may be taken to be identically zero for muonium and positronium since no polarizability contributions are expected, in contrast to the more complex situation which exists for hydrogen.) Since then, several additional papers have appeared which essentially confirm this estimate. Jensen, Kovesi-Domokos, and Schonberg [20.9], using a specific resonance model for inelastic e–p scattering and experimental inelastic e–p scattering data, find $\delta_N^{(2)} =$ 1 ppm. De Raphael [20.10], using similar data but rather general theoretical assumptions, has shown that one portion of $\delta_N^{(2)}$ lies within the bounds of −1.0 ppm and +2.7 ppm. More recently Gnädig and Kuti [20.11] have carried out a calculation very similar to de Raphael's and have essentially confirmed his result, finding bounds of −2.3 ppm and +3.2 ppm with an uncertainty of approximately ± 1 ppm on each bound. The remaining portion of $\delta_N^{(2)}$ has been estimated by Drell and Sullivan to be no more than ± 2 ppm. Jensen et al. [20.9] using their model of e–p scattering find +0.68 ppm for this portion. Gnädig and Kuti have also attempted to estimate this portion of $\delta_N^{(2)}$ and finally conclude that $\delta_N^{(2)}$ can be bounded by −6 ppm $<\delta_N^{(2)} <$ + 4 ppm. These workers further report that a more conservative approach to their calculation confirms this final result within a factor of 2.

The problem of how best to combine estimates of bounds which have built into them different amounts of conservatism with different estimates of accuracy is a difficult one. The available data appear to indicate that $\delta_N^{(2)}$ may more likely be positive than negative, but the value $\delta_N^{(2)} = 0$ is not inconsistent with any of the calculations. We shall therefore use for the polarizability correction for the proton the value

$$\delta_N^{(2)} = 0 \pm 3 \text{ ppm}, \quad (20.6)$$

where the assigned uncertainty is intended to represent the equivalent of one standard deviation.

We summarize the present status of the theory of the hyperfine structure in table 20.1. For the purposes of the present least-squares adjustment we can express the results as

$$\nu_{\text{Pshfs}} = \frac{7}{6} R_\infty c\alpha^2 \left(\frac{\mu_e}{\mu_B}\right)^2 [1-0.007131(57)] \text{ (58 ppm)}, \quad (20.7a)$$

TABLE 20.1. Summary of hyperfine structure correction terms

Term	Value		
	Ps	M	H
A	0.740810		
$\frac{\alpha}{\pi}R_1$	−0.003476	−0.0001793	−0.0000102
$\frac{\alpha}{\pi}\frac{a_p^2}{(1+a_p)}\cdot\frac{m_e}{m_p}R_2$			−0.0000240(9)
$(\alpha^2 \ln\frac{1}{\alpha})\cdot\frac{m_e m_x}{(m_e+m_x)^2}R_3$	0.000295	0.0000056	0.0000011
$\alpha^2 Q$	−0.000247(4)	−0.0001034(2)	−0.0001034(2)
Total, S	0.737382(4)	−0.0002771(2)	0.0001365(9)
Estimate for uncalculated terms[a]	±0.000100	±0.0000020	±0.0000005
$(1+\frac{3}{2}\alpha^2)(1+S)$	1.737521(100)	0.9998028(20)	0.9999434(10)

[a] Order of magnitude estimate of uncalculated recoil terms; see text.

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$$\nu_{\mathrm{Mhfs}} = \frac{16}{3} R_\infty c\alpha^2 \left(\frac{\mu_\mu}{\mu_p}\right)\left(\frac{\mu_p}{\mu_e}\right)\left(\frac{\mu_e}{\mu_B}\right)^2\left(1+\frac{m_e}{m_\mu}\right)^{-3} \times [1-0.0001972(20)]\ (2.0\ \mathrm{ppm}), \qquad (20.7b)$$

$$\nu_{\mathrm{Hhfs}} = \frac{16}{3} R_\infty c\alpha^2 \left(\frac{\mu_p}{\mu_e}\right)\left(\frac{\mu_e}{\mu_B}\right)^2\left(1+\frac{m_e}{m_p}\right)^{-3} \times [1-0.0000566(32)]\ (3.2\ \mathrm{ppm}). \qquad (20.7c)$$

Here and in the table we have used $\alpha^{-1} = 137.03602$ [0.1], $a_p = 1.7928$ (see sec. II.A.7), and g_e, g_μ, m_e/m_p, and m_e/m_μ as given in table 11.1. But it should be noted that the correction terms are relatively small and hence the actual values of the constants used are not critical. The uncertainties assigned eqs (20.7a) and (20.7b) arise primarily from estimates of the possible size of uncalculated higher order recoil terms [20.1] (see above). In keeping with the conservative approach we have taken throughout the present work, these estimates are more likely too large than too small. The uncertainty assigned eq (20.7c) arises primarily from estimates of the possible size of the proton polarizability contribution [eq (20.6)].

Finally, we point out that although the magnetic moment ratios μ_e/μ_B and μ_p/μ_e will be taken as auxiliary constants in the present adjustment (table 11.1), the ratio of the muon moment to proton moment, or equivalently, the muon-electron mass ratio which follows from it through the relation

$$\mu_\mu/\mu_p = \frac{(\mu_e/\mu_p)\, g_\mu}{(m_\mu/m_e)\, g_e}, \qquad (20.8)$$

is known only to an accuracy of a few ppm (see the following section) and must be taken to be an adjustable constant. Thus, in eqs (20.7a) and (20.7c), the measurement of the hyperfine splitting constitutes a measurement of α^2, while from eq (20.7b) one determines only $\alpha^2\mu_\mu/\mu_p$.

21. Ratio of the Magnetic Moment and Mass of the Muon to that of the Proton and Electron, μ_μ/μ_p and m_μ/m_e

To obtain a value of the fine structure constant from a measurement of the muonium hyperfine splitting requires knowledge of the two closely related quantities μ_μ/μ_p and $1 + m_e/m_\mu$. The relevant measurements of μ_μ/μ_p are summarized in table 21.1. The most accurate and comprehensive determination is that of a University of Washington-Lawrence Radiation Laboratory group (Crowe, Williams et al. [21.1]) who determined μ_μ/μ_p by stopping muons in various chemical environments in a magnetic field of 1.1T. The actual quantity determined is ω_μ^*/ω_p', the ratio of the muon to proton precession frequency, the former in the environment in question (indicated by the asterisk) and the latter for protons in H_2O. Varying the environment is of great importance since Ruderman [21.5] has proposed that the formation of the complex H_2O–μ^+–H_2O could reduce the muon shielding (compared with the 25.637 ppm for protons in H_2O) by 15 to 20 ppm. However, Crowe, Williams et al. in their determinations found no evidence of the Ruderman effect. Specifically, this was shown by measuring ω_μ^* in H_2O and NaOH. (In 0.1N NaOH, the μ^+ would be neutralized in $< 10^{-10}$s, suppressing the formation of the complex.) No significant difference was observed, nor was a significant difference observed when the muons were stopped in methylene cyanide, $CH_2(CN)_2$. The final result of all of their measurements is given as

$$\omega_\mu/\omega_p = \mu_\mu/\mu_p = 3.1833467(82)\ (2.6\ \mathrm{ppm}). \qquad (21.1)$$

In obtaining this value from ω_μ^*/ω_p', Crowe, Williams et al. took into account the small chemical shifts (i.e., shielding shifts) in ω_μ^* arising from the difference in zero point binding energies of proton and muon in the various possible molecular species which may be formed by the stopped muons (e.g., μHO, μH, etc.). They estimate (-1.8 ± 2.0) ppm in H_2O and NaOH, and $(+0.5 \pm 1.5)$ ppm in $CH_2(CN_2)$, all relative to protons in H_2O.

A result for μ_μ/μ_p obtained at the Princeton-Pennsylvania accelerator by essentially the same general method was reported by Hutchinson et al. in 1970 [21.2]:

$$\mu_\mu/\mu_p = 3.1833564(305)\ (9.6\ \mathrm{ppm}), \qquad (21.2)$$

where we have applied the (-1.8 ± 2.0) ppm correction of Crowe et al. to Hutchinson et al.'s original result: $\mu_\mu/\mu_p = 3.1833621(298)$ (9.4 ppm). Equation (21.2) agrees with an earlier and less accurate value obtained by Hutchinson and coworkers [21.6] at Columbia in the early 1960's: $\mu_\mu/\mu_p = 3.18338(4)$ (13 ppm). We do not use the Columbia result, however, since in our view the more reliable 1970 work replaces it.

Equations (21.1) and (21.2) are also consistent with the value of μ_μ/μ_p obtained at Chicago by DeVoe, Telegdi, et al. [21.3] from their double resonance—

TABLE 21.1. Summary of measurements of μ_μ/μ_p

Publication date and author	Value	Uncertainty (ppm)	Eq. No.
1972 Crowe, Williams, et al.[a]	3.1833467(82)	2.6	(21.1)
1970 Hutchinson et al.[b]	3.1833564(305)	9.6	(21.2)
1970 DeVoe, Telegdi, et al.[c] (Revised 1972, Jarecki and Herman[d])	3.1833496(148)	4.7	(21.5)

[a] Ref. [21.1]. [b] Ref. [21.2]. [c] Ref. [21.3]. [d] Ref. [21.4].

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"magic field" determination of the ground state hyperfine splitting in muonium. Their data yield

$$|g_j(\mathrm{M})/g'_\mu(\mathrm{M})| = 206.76509(84)\ (4.0\ \mathrm{ppm}), \quad (21.3)$$

where $g_j(\mathrm{M})$ is the electron g-factor in muonium and $g'_\mu(\mathrm{M}) = 2\mu_\mu(\mathrm{M})/\mu_B$. This result must now be corrected for bound state effects and the pressure shift in $g_j(\mathrm{M})$. The bound state correction to this ratio implied by the theory of Grotch and Hegstrom [7.2] (setting $a_p = a_\mu$) is $1 - 0.031 \times 10^{-6}$. The pressure shift correction, which is many times larger, has recently been calculated theoretically by Jarecki and Herman [21.4]. For the experimental conditions used in the Chicago experiment, they find that a (7.8 ± 2.3) ppm correction must be applied. These corrections yield

$$g_e/g'_\mu = 206.76670(96)\ (4.7\ \mathrm{ppm}), \quad (21.4)$$

which implies by means of the readily derived equation $\mu_\mu/\mu_p = (\mu_e/\mu_p)/(g_e/g'_\mu)$, and the value of μ_e/μ_p given in table 11.1,

$$\mu_\mu/\mu_p = 3.1833496(148)\ (4.7\ \mathrm{ppm}). \quad (21.5)$$

Although the validity of the Jarecki-Herman corrections may be open to some question due to the complexity of the problem [21.7], eq (21.5) is in fact in excellent agreement with the more accurate Crowe, Williams et al. result, eq (21.1).

As noted in the previous section, the ratio m_μ/m_e may be obtained from the relation

$$m_\mu/m_e = \frac{(\mu_e/\mu_p)\, g_\mu}{(\mu_\mu/\mu_p)\, g_e}. \quad (20.8)$$

While we will later take μ_μ/μ_p as an adjustable constant (i.e., let a best value for it be determined by our least-squares adjustment), we may obtain the factor $1+m_e/m_\mu$ to an accuracy such that it may be assumed to be an auxiliary constant by using the truncated value $\mu_\mu/\mu_p = 3.18335$ with a rather liberal 6 ppm uncertainty (2 in the last place). Using $g_e/2 = \mu_e/\mu_B$ and the other appropriate constants given in table 11.1, we find

$$m_\mu/m_e = 206.768(1), \quad (21.6)$$

which implies

$$1+m_e/m_\mu = 1.00483634(3)\ (0.03\ \mathrm{ppm}). \quad (21.7)$$

22. Ground-State Hyperfine Splitting in Muonium, Hydrogen, and Positronium: Experiment

Muonium

Two separate groups have had continuing programs to measure the ground-state hyperfine splitting ν_{Mhfs} in muonium (μ^+e^-) with the highest possible accuracy: Hughes and collaborators at Yale University;[16] and Telegdi and collaborators at the University of Chicago. The general principle of the experiment is straightforward. Polarized muons are stopped in a gas, either argon or krypton, and capture electrons to form muonium. In this process, the muon polarization is partially preserved and is reflected in the direction of the emitted decay positrons. A microwave field is applied and induces transitions between muonium Zeeman levels which involve spin-flip of the muon. By observing the change in the direction of the decay positrons associated with the spin flip, the (resonant) transition frequencies of interest can be determined.

One of the main difficulties associated with these experiments in the past has been how best to obtain ν_{Mhfs} from the values $\nu_{\mathrm{Mhfs}}(p)$ determined at the operating gas pressures, p. However, this "pressure shift" question has now been resolved [22.3]; it is generally agreed [22.4, 22.10] that for sufficiently high pressures ($p > \sim 10$ atm), a linear extrapolation to zero pressure is not adequate and that a quadratic term is required. That is, it must be assumed that

$$\nu_{\mathrm{Mhfs}}(p) = \nu_{\mathrm{Mhfs}}(0)[1 + ap + bp^2]. \quad (22.1)$$

Although the effect of collisions between the muonium and host gas atoms is called a pressure shift, it is actually a density effect. The data are by convention given in terms of that pressure which at 0 °C and assuming a perfect gas law would yield the actual density of the gas. The coefficient a is usually referred to as the fractional pressure shift or FPS.

With the above in mind, we very briefly summarize in table 22.1 and with the following comments the work carried out to date by both the Yale and Chicago groups.

(a) Yale. The first precision $\nu_{\mathrm{Mhfs}}(p)$ measurements were reported in 1964 by Cleland et al. [22.5]. Although originally analyzed using eq (22.1) with $b = 0$, in the final report on the experiment [22.6], the quoted preferred value is based on taking both a and b as free parameters.

In 1969, Thompson et al. [22.7] reported the results of significantly improved measurements carried out in weak ($\sim 3 \times 10^{-4}$T) and very weak ($\sim 10^{-6}$T) fields. A linear extrapolation to zero pressure gave the results shown in the table. The difference between the Ar and Kr values was attributed at the time to a possible nonlinearity in the argon pressure shift, that is, $b_{\mathrm{Ar}} \neq 0$.

These low field measurements were continued by the Yale group with improved techniques, and several interim reports have appeared [22.8 – 22.10]. A final paper analyzing all of the Yale low field data has now been prepared by Thompson et al. [22.11] who take both a and b as free parameters to obtain the results given. Their final low field value of ν_{Mhfs} is then obtained by taking a weighted average of the separate

[16] References [22.1] and [22.2] should be consulted for discussions of the earlier Yale muonium work.

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TABLE 22.1. Summary of published values of the ground-state hyperfine splitting in muonium, ν_{Mhfs}

Publication date	Magnetic field (T)	Stopping gas; and pressure in atm	Value (kHz)	Uncertainty (ppm)	a (10^{-9} torr^{-1})	b (10^{-15} torr^{-2})
			Yale Group			
1964[a]	~0.5	Ar; 9 to 64	4463150(60)	13	−4.05(49)	0
	Revised, 1972[b]		4463240(120)	27	−5.87(2.16)	35(40)
1969[c]	~0	Ar; 32 to 65	4463220(33)	7.4	−4.07(25)	0
		Kr; 20 to 43	4463302(27)	6.0	−10.4(3)	0
1973[d]	~0	Ar; 9 to 109	4463312(13)	2.9	−5.00(22)	8.1(2.5)
		Kr; 5 to 73	4463293(23)	5.2	−10.57(39)	8.6(5.9)
Weighted mean of 1973 Ar and Kr results			4463308(11)	2.5		
			Chicago Group			
1969[e]	1.13	Ar; 4.1 and 16.6	4463317(21)	4.7	−5.44(45)	0
	Revised, 1972[f]		4463313(18)	4.0	−5.27(40)	0
1970[g]	1.15	Ar; 4.0	4463293(23)	5.2	−5.44(45)[h]	0
		Kr; 3.4 and 15.6	4463304(10)	2.2	−10.50(32)	0
1971[i]	~0	Ar; 9.4	4463304.71(2.56)	0.6	−4.78(3)[j]	0
		Kr; 3.6 and 8.5	4463301.33(3.95)	0.88	−10.47(21)	0
1970 and 1971 Kr data together			4463301.17(2.3)	0.5	−10.37(70)	0

[a] Ref. [22.5]. [b] Ref. [22.6]. [c] Ref. [22.7]. [d] Ref. [22.11]. [e] Ref. [22.12]. [f] Ref. [22.13]. [g] Ref. [21.5].
[h] Assumed from 1969 measurements. [i] Ref. [22.14]. [j] Assumed hydrogen in argon value, ref. [22.15].

Ar and Kr results. The assigned uncertainty is primarily statistical; the total systematic uncertainty is only ~3kHz. Since the high field Ar measurements are much less precise than the low field measurements, they may be disregarded and this value of ν_{Mhfs} taken as the final result of all of the Yale work.

(b) Chicago. The first Chicago measurements were reported in 1969 by Ehrlich et al. [22.12]; and the final report on these early experiments has recently appeared [22.13]. They were carried out using a field independent transition at the "magic field" of 1.13 T. This enabled the use of larger gas target volumes and lower stopping pressures than would otherwise be possible.

The results of a second series of measurements by the Chicago group were reported in 1970 by DeVoe et al. [21.5]. (The data from this experiment were also used in the previous section to derive a value of μ_μ/μ_p). The quantity $\nu_{Mhfs}(p)$ was obtained from the frequencies of two separate transitions at the magic field used in the 1969 experiments. This "double resonance" technique has several advantages as outlined in ref. [21.5]. Two Kr data points and a single Ar point were obtained. ν_{Mhfs} was determined from the latter by extrapolating to zero pressure using the FPS determined from the Chicago group's 1969 argon work.

The results of the most recent series of Chicago experiments were reported by Favart et al. [22.14] in 1971, and are the most precise of all the muonium hfs measurements to date. This high accuracy is a direct consequence of the unique zero field "Ramsey resonance" method used. The two Kr data points and the single Ar point were found to yield the results given, where the latter was extrapolated to zero pressure using the FPS for the hydrogen hfs in argon as measured by Brown and Pipkin [22.15]. Since the 1971 and 1970 Kr data are in excellent agreement, Favart et al. fit them jointly to finally obtain a value of ν_{Mhfs} accurate to 0.5 ppm.

It should be noted that: (1) The Chicago data were generally obtained at sufficiently low pressures that the differences between purely linear fits and linear plus quadratic term fits are less than the uncertainties in the resulting values of ν_{Mhfs}. (2) The FPS values obtained from both the Yale and Chicago muonium measurements are in good agreement with the optical pumping values obtained by Ensberg and Morgan [22.16] from hydrogen, deuterium, and tritium hfs pressure shift measurements in argon and krypton:

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Argon

$$H : a = -4.803(15) \times 10^{-9}\ \text{torr}^{-1}, \quad (22.1a)$$

$$D : a = -4.803(65) \times 10^{-9}\ \text{torr}^{-1}, \quad (22.1b)$$

$$T : a = -4.785(15) \times 10^{-9}\ \text{torr}^{-1}, \quad (22.1c)$$

Krypton

$$H : a = -10.4(2) \times 10^{-9}\ \text{torr}^{-1}. \quad (22.1d)$$

These results, which are in agreement with but are more accurate than the similar measurements of Brown and Pipkin [22.15], show no evidence of a mass-dependent "isotope" effect. It may therefore be concluded that the muonium measurements are reasonably reliable, and that these "atomic" FPS values may be applied to muonium.

As far as our least-squares adjustment is concerned, we shall use the value

$$\nu_{\text{Mhfs}} = 4463303.82(1.80)\ \text{kHz}\ (0.40\ \text{ppm}), \quad (22.2)$$

as obtained by fitting all of the Chicago and low field Yale Ar and Kr data jointly with ν_{Mhfs}, a_{Ar}, a_{Kr}, b_{Ar}, and b_{Kr} as free parameters, that is, ν_{Mhfs} is constrained to be the same for both Ar and Kr. (The data used are summarized in table 22.2 and are taken directly from the references cited. However, we have revised the Chicago 3150 and 12600 torr Ar data points using our adopted value of $m_\mu/m_e = 206.768$ and the values of $g_e/2$ and $g_\mu/2$ given in table 11.1.) We further find $\chi^2 = 17.17$ for $32 - 5 = 27$ degrees of freedom, and

$$a_{\text{Ar}} = -4.817(70) \times 10^{-9}\ \text{torr}^{-1}, \quad (22.3a)$$

$$a_{\text{Kr}} = -10.595(66) \times 10^{-9}\ \text{torr}^{-1}, \quad (22.3b)$$

$$b_{\text{Ar}} = 6.28(91) \times 10^{-15}\ \text{torr}^{-2}, \quad (22.3c)$$

$$b_{\text{Kr}} = 8.30(1.39) \times 10^{-15}\ \text{torr}^{-2}. \quad (22.3d)$$

For the eight items of Chicago data, $\chi^2 = 3.74$; for the 24 items of Yale data, $\chi^2 = 13.42$. Although one might argue that some of the Yale data could be grouped together because they were obtained at very nearly the same pressures, we retain each measurement as a separate item because they are all experimentally independent. Clearly, all of the data are in agreement and the resulting FPS values are consistent with the atomic values, eq (22). We have chosen to handle the Yale and Chicago data jointly in this manner because we believe that overall, it is the most self consistent way of doing so.

It is of interest for purposes of comparison to derive here a value of the fine-structure constant from the Yale-Chicago measurement of the muonium hyperfine splitting. Using eqs (22.2) and (20.7b), the appropriate auxiliary constants of table 11.1, and the value $\mu_\mu/\mu_p = 3.1833479(70)$ (2.2 ppm) obtained from the weighted average of the three values given in table 21.1, we find

$$\alpha^{-1}(\text{Mhfs}) = 137.03634(21)\ (1.5\ \text{ppm}). \quad (22.4)$$

Hydrogen

The experimental data on hyperfine structure includes the most accurate physical measurement known—the hydrogen maser measurement of the hydrogen hyperfine splitting, ν_{Hhfs}. The two most recent determinations of this frequency are those of Hellwig et al. [22.17] who report

$$\nu_{\text{Hhfs}} = 1420405751.7691(24)\ \text{Hz (Experiment 1)}, \quad (22.5a)$$

$$\nu_{\text{Hhfs}} = 1420405751.7667(18)\ \text{Hz (Experiment 2)}, \quad (22.5b)$$

and of Essen et al. [22.18] who give

$$\nu_{\text{Hhfs}} = 1420405751.7667(10)\ \text{Hz}. \quad (22.5c)$$

These three measurements are in excellent agreement but they are approximately 0.02 Hz less than the value obtained by Vessot et al. which was used by Taylor et al. [0.1]. Although the difference is some 12 times the standard deviation assigned the latter value, the change is entirely negligible as far as our present adjustment is concerned.

The value of the fine-structure constant implied by eqs (20.6), (20.7c), (22.5), and the auxiliary constants of table 11.1, is

TABLE 22.2. Summary of measurements of $\nu_{\text{Mhfs}}(p)$ by the Chicago and Yale groups used in the present work to determine ν_{Mhfs}

Argon		Krypton	
Pressure (torr)	Value (kHz)	Pressure (torr)	Value (kHz)
Chicago			
3030[a]	4463220(22)	2598[a]	4463182.4(5.0)
3150[b]	4463249.3(10.7)	2742[c]	4463173.26(1.90)
7150[c]	4463152.17(2.37)	6469[c]	4462999.78(2.96)
12600[b]	4463027.6(10.4)	11830[a]	4462750.4(2.2)
Yale[d]			
7100	4463162(10)	4328	4463066(30)
10392	4463089(11)	7292	4462906(40)
24306	4462754(32)	7328	4462975(34)
24487	4462783(30)	7349	4462998(59)
24487	4462752(32)	7581	4462921(33)
24528	4462803(30)	15876	4462578(38)
24720	4462786(38)	15928	4462578(25)
26685	4462746(31)	15928	4462558(36)
46104	4462348(33)	15959	4462554(26)
49051	4462341(22)	16057	4462558(23)
82640	4461718(12)	32854	4461782(16)
		33951	4461719(21)
		55200	4460809(11)

[a] Ref. [21.3]. [b] Ref. [22.13], and see text. [c] Ref. [22.14]. [d] Ref. [22.11].

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$$\alpha^{-1}(\text{Hhfs}) = 137.03597(22)\ (1.6\ \text{ppm}). \qquad (22.6)$$

The difference between the muonium and hydrogen hyperfine splitting α values, eqs (22.4) and (22.6), is thus (2.7 ± 2.2) ppm. If one were to ascribe real significance to this difference, it would have to arise from inadequacies of the hyperfine splitting theory of section II.C.20 or from a nonzero value of the proton polarizability contribution, $\delta_N^{(2)}$. Indeed, additional information concerning this quantity may be found from the ratio $\nu_{\text{Mhfs}}/\nu_{\text{Hhfs}}$. Equations (20.7b) and (20.7c) give

$$\frac{\nu_{\text{Mhfs}}}{\nu_{\text{Hhfs}}} = \frac{\mu_\mu}{\mu_p}\left(\frac{1+m_e/m_p}{1+m_e/m_\mu}\right)^3 \times \frac{0.9998594(23)}{1+\delta_N^{(2)}}, \qquad (22.7)$$

where we have now omitted the 3 ppm uncertainty assigned in section II.C.20 to $\delta_N^{(2)}$. From eqs (22.2), (22.5) and the above weighted average value of μ_μ/μ_p, we obtain

$$\delta_N^{(2)} = (5.5 \pm 3.2)\ \text{ppm}. \qquad (22.8)$$

Although this result is not inconsistent with the value (0 ± 3) ppm adopted in section II.C.20, it is clear that further work is needed in this area.

Positronium

For completeness, although we shall not use the result in our least squares adjustment, we note that the experimental value of the hfs in positronium, ν_{Pshfs}, measured by Carlson, Hughes, et al. at Yale [22.19]

$$\nu_{\text{Pshfs}} = 203396(5)\ \text{MHz}\ (25\ \text{ppm}), \qquad (22.9)$$

has an accuracy which is less than an order of magnitude below what would be required to make it eligible for inclusion in an adjustment. (This value replaces the earlier result of the Yale group reported by Theriot et al. [22.20], $\nu_{\text{Pshfs}} = 203403(12)$ MHz (60 ppm).)

The positronium value of the fine-structure constant implied by eqs (22.9) and (20.7a) is

$$\alpha^{-1}(\text{Pshfs}) = 137.0374(43)\ (31\ \text{ppm}), \qquad (22.10)$$

where most of the uncertainty comes from the theoretical uncertainties in eq (20.7a) (29 ppm, to be combined quadratically). If terms of order α^2 are neglected (as well as the fourth order vacuum polarization term $(-\alpha^2/4)\ln\frac{1}{\alpha}$ discussed in section II.C.20), one can write [20.4]

$$\nu_{\text{Pshfs}} = \alpha^2 R_\infty c\left[\frac{7}{6} - \frac{\alpha}{\pi}\left(\frac{16}{9} + \ln 2\right) + \frac{3}{4}\alpha^2 \ln\frac{1}{\alpha}\right], \qquad (22.11)$$

from which we find

$$\alpha^{-1}(\text{Pshfs}) = 137.0426(17)\ (13\ \text{ppm}), \qquad (22.12)$$

where no allowance is made for the unstated and uncalculated theoretical terms. (If the new vacuum polarization is included, α^{-1}(Pshfs) becomes 137.0387(17) (13 ppm).) Clearly positronium will be an important source of information on the fine-structure constant only when the theoretical expression has been extended to include *all* terms through order α^2.

23. Fine-Structure

We consider here only those measurements which yield a value of α with an uncertainty of less than 5 ppm since the uncertainties in the values of α derivable from measurements of $2e/h$ and γ_p', ν_{Hhfs}, and ν_{Mhfs}, are of order 1.5 ppm or less. (But we neglect the measurements of Lamb and coworkers carried out in the early 1950's [0.1] because in our view, the newer determinations of the 1960's replace them.) Furthermore, Erickson [23.1, 23.2] has refined the theory of the energy levels in hydrogen-like atoms to the point where the uncertainty in the theoretical expression for the Lamb shift in hydrogen ($\mathcal{S}_{\text{H}}$) for $n=2$ ($2S_{1/2}-2P_{1/2}$ interval) is only 0.0102 MHz [23.2][17] (aside from that due to the uncertainty in the numerical value of α), while the experimental determinations of $\mathcal{S}_{\text{H}}$ have uncertainties several times larger [0.1]. It is therefore now more accurate to use the theory of the Lamb shift in combination with the highly accurate theory of the fine-structure splitting in hydrogen (ΔE_{H}) for $n=2$ ($2P_{3/2}-2P_{1/2}$ interval), to obtain a theoretical expression for $(\Delta E-\mathcal{S})_{\text{H}}$, $n=2$ ($2P_{3/2}-2S_{1/2}$ interval), and to calculate values of α from this expression and the several experimental determinations of $(\Delta E-\mathcal{S})_{\text{H}}$. (The alternate but less accurate procedure would be to combine experimental values of $\mathcal{S}_{\text{H}}$ and $(\Delta E-\mathcal{S})_{\text{H}}$, and to then calculate α from the theoretical expression for ΔE_{H}.)

Thus, we shall make no real use of experimental Lamb shift values except to note that they are in good agreement with theory (see refs. [19.1, 23.1, 23.3, 23.4]), thereby giving some assurance that the theory of the Lamb shift is well in hand. This situation is in marked contrast to that which existed at the time of the 1969 review of Taylor et al. [0.1] and is primarily due to the discovery by Appelquist and Brodsky [23.5] that Soto's [23.6] earlier calculation of the fourth order radiative correction to $\mathcal{S}$ was in error.

The contribution of the one-photon electron self-energy to the energy levels of hydrogen-like atoms may be expressed as [23.1] [18]

$$\Delta E_n = \frac{8\alpha^3}{3\pi n^3} Z^4 R_\infty c[C_{41}\ln(Z\alpha)^{-2} + C_{40} + H(Z\alpha)], \qquad (23.1a)$$

[17] We should like to thank Professor Erickson for providing us with his most recent results regarding the theory of the fine-structure of hydrogen-like atoms.

[18] This paper should be consulted for references to earlier work.

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where for $(Z\alpha)<<1$ (roughly $Z<30$), $H(Z\alpha)$ may be well approximated by the series

$$H(Z\alpha) = C_5(Z\alpha) + [C_{62}\ln^2(Z\alpha)^{-2} + C_{61}\ln(Z\alpha)^{-2} + C_{60}](Z\alpha)^2 + C_7(Z\alpha)^3 + \ldots \quad (23.1b)$$

The coefficients C_{41}, C_{40}, C_5, C_{62}, and C_{61} have been previously computed as functions of the quantum numbers n, l, and j. Erickson [23.1] has now obtained an improved approximation for $H(Z\alpha)$ which is valid for all $Z\alpha$, and has also evaluated the corresponding approximate values for the coefficients C_{60} and C_7. Since experimental fine-structure data of sufficient accuracy to be of interest for deriving values of α are primarily restricted to the $n=2$ levels of hydrogen, we need only the values for six specific coefficients. Erickson gives [23.2]:

$$\begin{aligned} C_{60}(2P_{1/2}) &= -0.352, & C_7(2P_{1/2}) &= -1.28, \\ C_{60}(2S_{1/2}) &= -17.598, & C_7(2S_{1/2}) &= 18.48, \\ C_{60}(2P_{3/2}) &= -0.438, & C_7(2P_{3/2}) &= -0.32, \end{aligned} \quad (23.2)$$

where the one standard deviation uncertainty in $H(Z\alpha)/(Z\alpha)^2$ is estimated to be ± 0.5 for S states, ± 0.33 for $2P_{1/2}$ states, and ± 0.18 for $2P_{3/2}$ states.

Taking into account the $C_{60}(Z\alpha)^2$ and $C_7(Z\alpha)^3$ terms of eq (23.1b), using the numerical values of eq (23.2) for the coefficients C_{60} and C_7, and taking $\alpha^{-1}=137$, leads to the following expression for the fine-structure splitting in hydrogen, $n=2$:[19]

$$\Delta E_{\mathrm{H}}(n=2) = \frac{R_\infty c\alpha^2}{16}\left(1+\frac{m_e}{m_p}\right)^{-1}\left[g_e\left(1+\frac{m_e}{m_p}\right)^{-1} - \left(1+\frac{m_e}{m_p}\right)^{-2} + \frac{5}{8}\alpha^2 - \frac{\alpha^3}{\pi}\left(\ln\alpha^{-2} + 0.421 \pm 1.5\right)\right]. \quad (23.3)$$

The total QED contribution (with the exception of the electron anomalous moment) is represented by the last term in this equation and amounts to only 1.2 ppm. The quantity 0.421 ± 1.5 in this term is Erickson's higher-order QED contribution as represented by C_{60} and C_7 and is given by

$$0.421 = -\frac{16}{3}[C_{60}(2P_{3/2}) - C_{60}(2P_{1/2}) + \alpha C_7(2P_{3/2}) - \alpha C_7(2P_{1/2})];$$

it amounts to only 0.0006 MHz (0.05 ppm). The ± 1.5 uncertainty in the theoretical expression for ΔE_{H}, $n=2$ [eq (23.3)] (not including that due to α), is that given by Erickson [23.2] and corresponds to 0.0021 MHz (0.19 ppm). It includes only the uncertainty in $H(Z\alpha)$ due to a possibly large QED contribution from the S-state component of the relativistic $2P_{1/2}$ state since the remaining uncertainties in the $2P_{1/2}$ and $2P_{3/2}$ states cancel.

The theoretical expression for the Lamb shift may be obtained from that given by Taylor et al. [ref. [0.1], eq (138a)] by making the appropriate modifications required by the revision of Soto's expression for the fourth order radiative correction, and by Erickson's new work [23.1, 23.2]. The former may be included by replacing the coefficient m in eq (138a) of ref. [0.1] by the exact analytic expression derived by Peterman [23.3] (its numerical value is $m=2\times 0.46994\ldots=0.93988\ldots$). The major features of Erickson's new calculations may be included by making the following three modifications [23.2];

(1) The term -0.3285 becomes $-0.3285 + 1.285 \times (\alpha/\pi)$, that is, the $(\alpha/\pi)^3$ term is to be included in the expression for the electron anomalous moment (see sec. II.C.19).

(2) For hydrogen, the nuclear structure term must be multiplied by an overall correction factor of 0.996 ± 0.002 to take into account additional nuclear structure effects.

(3) For $n=2$, the previously estimated term $-(4\pi^2/3+4+4\ln^2 2)$ plus a previously uncalculated term is replaced by

$$[C_{60}(2S_{1/2}) - C_{60}(2P_{1/2})] + \alpha[C_7(2S_{1/2}) - C_7(2P_{1/2})].$$

The uncertainty in the theoretical expression for $\mathcal{S}_{\mathrm{H}}$, $n=2$, is given by Erickson as 0.0102 MHz (9.7 ppm) (not including the uncertainty in α), and is the RSS of the 0.0099 MHz and 0.0026 MHz respective uncertainties in the $2S_{1/2}$ and $2P_{1/2}$ levels.

The theoretical expression for $(\Delta E - \mathcal{S})_{\mathrm{H}}$, $n=2$, is simply the difference between that for ΔE_{H} and $\mathcal{S}_{\mathrm{H}}$. The uncertainty in this theoretical expression is calculated directly from the RSS of the respective 0.0099 MHz and 0.0014 MHz uncertainties in the $2S_{1/2}$ and $2P_{3/2}$ levels. Thus, the 0.0021 MHz uncertainty in the $2P_{1/2}$ level due to the uncalculated contribution from the S-state component of the relativistic $P_{1/2}$ state (see above) is taken as part of the uncertainty in $\mathcal{S}_{\mathrm{H}}$, all of the uncertainty in ΔE_{H}, but none of the uncertainty in $(\Delta E - \mathcal{S})_{\mathrm{H}}$.

For informational purposes, we note that our theoretically predicted values for ΔE_{H}, $\mathcal{S}_{\mathrm{H}}$, and $(\Delta E - \mathcal{S})_{\mathrm{H}}$, $n=2$, using the auxiliary constants of table 11.1 and taking $\alpha^{-1}=137.03602$, exactly, are

$$\Delta E_{\mathrm{H}}(n=2) = 10969.0348(21)\ \text{MHz (0.19 ppm)},$$

$$\mathcal{S}_{\mathrm{H}}(n=2) = 1057.9158(102)\ \text{MHz (9.7 ppm)},$$

$$(\Delta E - \mathcal{S})_{\mathrm{H}}(n=2) = 9911.1190(100)\ \text{MHz (1.0 ppm)}.$$

Table 23.1 summarizes the fine-structure measurements to be considered herein. The implied values of α have been obtained from the experimental results and the theoretical expressions for ΔE_{H} and $(\Delta E - \mathcal{S})_{\mathrm{H}}$

[19] There is some variation in the literature in the way the reduced mass factors are written since the exact solution of the relativistic two-body problem has yet to be obtained. However, the differences between the various expressions are only on the order of $\alpha^2(m_e/m_p)$ or a few parts in 10^8 (see ref. [0.1]).

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TABLE 23.1. Summary of fine-structure measurements and implied values of α. (For comparison purposes, the other high precision QED α values discussed have been included.)

Publication date and author	Quantity measured	Value (MHz, except a_e)	Uncertainty (ppm)	Implied value of α^{-1}	Uncertainty (ppm)	Eq. No.
1972, Baird et al.[a]	ΔE_H	10969.127(87)	7.9	137.03544(54)	3.9	(23.4)
1971, Kaufman, Lamb, et al.[b]	$(\Delta E-\mathcal{S})_H$	9911.377(26)	2.6	137.03416(20)	1.5	(23.5)
1971, Shyn et al.[c]	$(\Delta E-\mathcal{S})_H$	9911.250(63)	6.4	137.03508(46)	3.3	(23.6)
1970, Cosens and Vorburger[d]	$(\Delta E-\mathcal{S})_H$	9911.173(42)	4.2	137.03563(31)	2.3	(23.7)
1971, Kponou, Hughes, et al.[e]	ν_{01} (He)	29616.864(36)	1.2	137.03595(42)	3.1	(23.8)
1971, Wesley and Rich[f] (revised 1972, Granger and Ford[g])	a_e	0.0011596567(35)	3.0	137.03563(42)	3.1	(19.4)
1971, 1973, Yale and Chicago groups[h]	ν_{Mhfs}	4463.3038(18)	0.40	137.03634(21)	1.5	(22.4)
1970, Hellwig et al.[i], 1971, Essen et al. (wtd. mean)	ν_{Hhfs}	1420.4057517670(8)	0.6×10^{-6}	137.03597(22)	1.6	(22.6)

[a] Refs. [23.7, 0.1]. [b] Ref. [23.8]. [c] Ref. [23.9]. [d] Ref. [23.10]. [e] Ref. [23.11]. [f] Ref. [6.1]. [g] Ref. [6.2]. [h] Refs. [22.11, 22.14], and see text. The uncertainty in ν_{Mhfs} (theory) was taken to be 2 ppm as discussed in the previous section. [i] Refs. [22.17, 22.18].

as just discussed. In each case, the assigned uncertainty is the RSS of the final uncertainty assigned the experiment (column 4) and the uncertainty in the theoretical expression used.[20] The actual dependence on α of the theoretical expression was also taken into account. (In eq (23.3), the theoretical relation of sec. II.C.19 was used for g_e.) The helium fine-structure α value will be discussed below. For informational purposes, we have also included in table 23.1 the high precision values of α derived in sections II.C.19 through 22. Thus, the table conveniently compares all of the QED data which might possibly be used in our adjustment. The following comments apply as well.

(a) ΔE_H, Baird et al. The analysis of this level-crossing experiment as given in ref. [0.1] remains unchanged with the exception that the uncertainty must be decreased somewhat. This is due to a decrease in the uncertainty assigned to the nonlinearity of the electronics in Baird et al.'s final report on the experiment [23.7]. The statistical standard deviation of the 84 runs comprising this determination is 62 ppm; of the mean, 6.8 ppm.

(b) $(\Delta E-\mathcal{S})_H$, Kaufman, Lamb, et al. The final result of this microwave-optical experiment quoted by Kaufman, Lamb, et al. and given in the table is the weighted mean of the results obtained from measurements on the αa and αb transitions:

$$\alpha a\text{: } 9911.363(31) \text{ MHz } (3.1 \text{ ppm}); \quad (23.4a)$$

$$\alpha b\text{: } 9911.407(45) \text{ MHz } (4.5 \text{ ppm}). \quad (23.4b)$$

The statistical standard deviations of the 148 αa runs and 62 αb runs were 22 ppm and 7.9 ppm, respectively; of their means, 1.8 ppm and 1.0 ppm. Thus, as in the Baird et al. ΔE_H experiment, a large number of measurements were used to compensate for a great deal of scatter in the data.

(c) $(\Delta E-\mathcal{S})_H$, Shyn et al. Both the β^+b^+ and β^+d^+ transitions were measured in this atomic beam experiment:[21]

$$\beta^+b^+\text{: } 9911.255(59) \text{ MHz } (6.0 \text{ ppm}), \quad (23.5a)$$

$$\beta^+d^+\text{: } 9911.242(90) \text{ MHz } (9.1 \text{ ppm}), \quad (23.5b)$$

where the uncertainty is statistical only. Since the histogram of the 139 measurements carried out on these two transitions showed no clustering, Shyn et al.

[20] The slight correlation between α values derived from the three $(\Delta E-\mathcal{S})_H$ measurements which is introduced through the common 1.0 ppm theoretical uncertainty is sufficiently small that it may be ignored.

[21] Note that the preliminary results of this experiment and the following one of Cosens and Vorburger as given in ref. [0.1] differ from the final values quoted here. However, the general comments on these experiments made therein remain valid. This reference should also be consulted for additional discussions of the Baird et al. and Kaufman, Lamb, et al. experiments.

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combined them together to obtain the final value quoted in the table. The statistical standard deviation of the 139 measurements was 62 ppm; of the mean, 5.2 ppm. Again we have a situation in which the effect of a large experimental scatter is reduced by a large number of measurements.

(d) $(\Delta E - \mathcal{S})_H$, Cosens and Vorburger. The following results for $(\Delta E - \mathcal{S})_H$ were obtained from the four transitions measured in this experiment:

$$\beta b^-: 9911.281(92) \text{ MHz } (9.3 \text{ ppm}), \quad (23.6a)$$

$$\beta b^+: 9911.144(85) \text{ MHz } (8.6 \text{ ppm}), \quad (23.6b)$$

$$\beta d^-: 9911.196(76) \text{ MHz } (7.7 \text{ ppm}), \quad (23.6c)$$

$$\beta d^+: 9911.084(84) \text{ MHz } (8.5 \text{ ppm}). \quad (23.6d)$$

The statistical uncertainty in the mean of each was, respectively 7.5, 7.7, 7.4, and 6.6 ppm. Thus, as in all of the fine-structure measurements discussed so far, the systematic uncertainties are estimated to be small relative to the random uncertainties. The final result quoted by Cosens and Vorburger is as given in the table and was obtained from the weighted mean of these four values. The fact that the Birge ratio for these is 0.95 would seem to indicate that the uncertainties are realistic. But again, a large number of measurements for each transition has been used to significantly reduce the effect of a large random scatter.

(e) $2\,^3P_0 - 2\,^3P_1$ fine-structure interval in atomic helium. This interval, referred to as ν_{01}, has been measured to an accuracy of 1.2 ppm by Hughes and collaborators [23.11] at Yale using an atomic beam-optical-microwave method. The value quoted is the weighted mean of the results obtained from 73 separate resonance curves. The final uncertainty is the RSS of the 0.035 MHz statistical uncertainty of this mean and the 0.007 MHz systematic uncertainty assigned the so called slope correction. Unfortunately, the theory of the fine-structure in atomic helium has not reached the point where full advantage of this 1.2 ppm accuracy may be taken. The present state of the theory has been summarized recently by Daley et al. [23.12] and was motivated by their own recent contributions to the problem. Using their results and the values of c, R_∞, and $1 + m_e/m_\alpha$ given in table 11.1, we find that the Yale experimental value for ν_{01} yields the value of α given. The uncertainty is due almost entirely to the 6 ppm uncertainty in ν_{01} (theory).

Although this result has an assigned uncertainty comparable to that of the other α values listed in table 23.1, we shall not consider it for possible inclusion in our adjustment because many of the terms in the theory require complex numerical calculations, and a consistent evaluation of the accuracy of these terms is extremely difficult. At present, the 6 ppm uncertainty can only be considered as a very approximate estimate. Thus, the agreement between theory and experiment should be viewed as a verification of the theoretical calculations rather than a determination of the numerical value of the fine-structure constant.

Table 23.1 reveals that the Kaufman, Lamb, et al. result, although assigned the lowest uncertainty, appears to be inconsistent with the other data. These workers were aware of this situation and in their final report on the experiment [23.8] consider various possibilities which might account for the apparent error. Unfortunately, they could not resolve the discrepancy. But it was noted that the apparent linear magnetic field dependence of the various fine-structure measurements pointed out by Kaufman [23.13] might imply some insufficiency of the theory of the Zeeman effect.

In a different vein, we would like to reiterate here what was pointed out by Taylor et al. [0.1], namely, that in relatively low precision experiments such as all of the fine-structure measurements just discussed, it is not feasible to *experimentally* investigate possible sources of systematic error of a size approaching the statistical standard deviation of the mean of the measurements. One must rely primarily on theoretical estimates of such effects. On this basis, it seems to us that unless an experimenter has very strong *a priori* reasons to believe that his experimental scatter is indeed purely random (i.e., he is doing essentially a counting experiment), then he is fooling himself if he quotes a final uncertainty which is less than one third to one fourth the statistical standard deviation of his measurements. If this criterion were to be applied to the experiments at hand, any and all discrepancies would immediately disappear. However, for the purposes of investigating the overall compatibility of the stochastic data to be considered for inclusion in our adjustment, we shall retain the original uncertainties assigned by the experimenters.

D. Other Less Precise Quantities

Here we very briefly discuss three quantities with relatively large uncertainties and which, although of great intrinsic importance, play no role as yet in a least-squares adjustment of the constants. They are the Newtonian gravitational constant, G; the molar volume of an ideal gas at s.t.p., V_m; and the Stefan-Boltzmann constant, σ. This situation may, of course, change in the future with advances in both experiment and theory. We also summarize here all of the stochastic data to be considered for possible inclusion in our adjustment.

24. Newtonian Gravitational Constant, G

At the present time, there exists no verified theoretical equation relating G to any other physical constant. Thus, it can have no direct bearing on the output values of our adjustment. Our aim here is simply to

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TABLE 24.1. Summary of recent high precision measurements of G

Publication date and author	Method	Value (10^{-11} $m^3 \cdot s^{-2} \cdot kg^{-1}$)	Uncertainty (ppm)	Eq. No.
1930, Heyl[a]	Torsion balance (oscillation)	6.6721(73)	1090	(24.1)
1942, Heyl and Chrzanowski[b]	Same	6.6720(49)	740	(24.2)
1969, Rose, Beams et al.[c]	Accelerating table	6.674(3)	450	(24.3)
1972, Pontikis[d]	Torsion balance (resonance)	6.67145(10)	15	(24.4)

[a] Ref. [24.1]. [b] Ref. [24.2]. [c] Ref. [24.3]. [d] Ref. [24.4]. The uncertainty quoted is the statistical standard deviation of the mean; see text.

decide on the basis of the available measurements what is the current best value of G. For this purpose, we consider only the four most recent and precise determinations. These are summarized in table 24.1.

(a) Heyl. In 1930, Heyl [24.1] reported the results of his 1925 to 1928 oscillating torsion balance measurements at NBS in which he used three different materials for the small spherical masses (balls). He found for G, in units of 10^{-11} $m^3 \cdot s^{-2} \cdot kg^{-1}$:

Gold	Platinum	Glass
6.6782(16),	6.6640(13),	6.6740(12),

where the quoted uncertainties are the statistical standard deviations of the means of the five individual measurements which comprised each determination. Unfortunately, Heyl was unable to explain the discrepancy between the platinum value and the gold and glass values. Using these uncertainties one finds a Birge ratio for the weighted mean of the three measurements of $R_B = 5.2$, or a value of χ^2 of 53 with 2 degrees of freedom. The assigned uncertainties therefore have little significance and we instead use an unweighted mean and obtain

$$G = 6.6721(73) \times 10^{-11}\ m^3 \cdot s^{-2} \cdot kg^{-1}, \qquad (24.1)$$

where the quoted uncertainty is the standard deviation of the observations. (Since the three measurements are not consistent, at least one of them contains an unidentified systematic error. The standard deviation of the distribution reflects this, whereas the standard deviation of the mean would imply that the existing systematic errors actually had zero mean.)

(b) Heyl and Chrzanowski. In 1940 Heyl and Chrzanowski repeated the 1930 experiment with several improvements in apparatus and technique [24.2]. Using platinum balls and both annealed and hard drawn tungsten torsion fibers, they found unexplained systematic differences. From sets of 5 separate determinations using each type of fiber they found

Annealed	Hard Drawn
6.67554(51),	6.66854(83).

In this case the calculated value of χ^2 is 52 (1 degree of freedom). Hence we shall again use the unweighted mean

$$G = 6.6720(49) \times 10^{-11}\ m^3 \cdot s^{-2} \cdot kg^{-1}. \qquad (24.2)$$

Despite the internal inconsistencies both in these measurements and in the 1930 measurements, the means of the two series are in surprisingly good agreement.[22]

(c) Rose, Beams, et al. In 1969, Beams and his collaborators [24.3] at the University of Virginia reported the first results of their entirely new method for determining G which, among other things, requires measuring the acceleration of a rotating table. Their quoted result is

$$G = 6.674(3) \times 10^{-11}\ m^3 \cdot s^{-2} \cdot kg^{-1}, \qquad (24.3)$$

where the assigned uncertainty corresponds to one standard deviation and is statistical only. Since their original experiments, the University of Virginia group has continued to refine its apparatus and to investigate possible sources of systematic error. Several problem areas have been identified and discussed [24.6, 24.7].

(d) Pontikis. The most recent and superficially most precise determination of G to date would appear to be that recently reported in a short letter by Pontikis [24.4]. Using the resonance-torsion balance method with silver, copper, bronze, and lead balls, he finds from ten measurements for each material carried out during May, 1971:

Silver	Copper	Bronze	Lead
6.67162(17),	6.67157(17),	6.67122(21),	6.67126(22),

[22] An interesting analysis of the time dependence of the Heyl, and Heyl and Chrzanowski measurements is given by Stephenson [24.5].

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where the uncertainties are the statistical standard deviations of the means of each series of ten measurements. The weighted mean of these four values is

$$G = 6.67145(10) \times 10^{-11}\ \mathrm{m^3 \cdot s^{-2} \cdot kg^{-1}}. \quad (24.4)$$

The uncertainty, which is statistical only, is based on external consistency but R_B is only 1.1. Unfortunately, insufficient information is given in Pontikis' short note to properly evaluate his systematic error. Indeed, he obtained a significantly different result from measurements taken in September, 1971 [24.4].

We shall adopt in this work the weighted mean of the Heyl, and Heyl and Chrzanowski results, eqs (24.1) and (24.2):

$$G = 6.6720(41) \times 10^{-11}\ \mathrm{m^3 \cdot s^{-2} \cdot kg^{-1}}\ (615\ \mathrm{ppm}). \quad (24.5)$$

(These two experiments may be considered independent since they differed significantly in apparatus and technique.) We do not include the Rose, Beams, et al. result because the experiment is still underway and several disturbing systematic effects have been uncovered since the original publication [24.6, 24.7]. Similarly, we do not consider the Pontikis result at this time even though its claimed precision is far superior to that of the other three because the value quoted is based on only a small portion of Pontikis' total data. He indicates [24.4] that a paper giving the results of 1000 measurements of G, analyzed as a function of time of year and material, will soon be forthcoming.

25. Molar Volume of an Ideal Gas, V_m, and the Molar Gas Constant R

The equation of state of a perfect or ideal gas is $pV = RT$; for any real gas one has

$$pV = RT[1 + B(T)/V + C(T)/V^2 + \ldots],$$

where B, C, . . . are the virial coefficients. The gas constant R can be found by measuring the pressure of a gas at different molar volumes, and extrapolating the pV product to zero pressure (infinite volume). It also follows that the product

$$V_m = RT_0/p_0, \quad (25.1)$$

where $T_0 \equiv 273.15$ K is the thermodynamic temperature[23] corresponding to 0 °C [25.1], is the molar volume of a perfect gas at standard conditions (t = 0 °C, p_0 = 1 atm[24]).

Several extremely careful absolute gas density experiments were carried out by a number of workers during the period 1924 to 1941. Because the molecular weight of O_2 was exactly 32 on the old chemical scale of atomic weights, this gas primarily was used for these measurements. Batuecas has summarized and reviewed the relevant data in two separate publications [25.3, 25.4]. The final result recommended in his most recent review, using 1 litre = 1000.028(4) cm^3,[25] becomes

$$V_m = 22413.83 \pm 0.70\ \mathrm{cm^3 \cdot mol^{-1}}\ (31\ \mathrm{ppm}). \quad (25.2)$$

Batuecas, however, has not utilized any of the more recent data on the compressibility of gases, which yield values of the virial coefficients [25.5], to increase the accuracy of the extrapolation of the absolute density measurements to zero pressure. This extrapolation, for O_2 or N_2, can probably be assigned an uncertainty of no better than 10 ppm. It is probably not productive to push this accuracy further since other sources of error are of the same magnitude. More difficult is the accurate determination of pressure; in a mercury manometer one must not only be able to measure the height of the mercury column to an accuracy of better than 10 μm but must know its purity (and density) to better than 10 ppm. In order to know the molecular weights of a gas such as O_2 or N_2 to 10 ppm requires a determination of the isotopic composition to an accuracy of 0.01 percent.

The quoted uncertainty in eq (25.2) may thus well represent the limit to be achieved in this type of experiment even if isotopically separated gases are utilized. An alternative approach which is being pursued [25.6, 25.7] is the accurate measurement of the velocity of sound. Since the determination of sound velocity, $\bar{c}$, from

$$\bar{c}^2 = \gamma RT/M, \quad (25.3)$$

where γ is the specific heat ratio and M the molecular weight of the gas, does not require the difficult determination of the pressure in absolute units, this experiment may be able to yield an accuracy approaching a few parts per million.

For the present adjustment we shall accept Batuecas' value, eq (25.2). From this one then has

$$R = 82.0568(26)\ \mathrm{cm^3 \cdot atm \cdot mol^{-1} \cdot K^{-1}}$$
$$= 8.31441(26)\ \mathrm{J \cdot mol^{-1} \cdot K^{-1}} \quad (25.4)$$

It should be noted that the gas constant R is related to the Boltzmann constant k by the equation

$$k = R/N_A. \quad (25.5)$$

[23] The kelvin, K, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. The celsius temperature scale is defined by t(°C) = T − 273.15. The Celsius scale is *not* a "centigrade" temperature scale; the ice point and steam point (at 1 atmosphere) are 0 ± 0.0001 °C and 99.996 ± 0.003 °C, respectively [25.2]. It should be noted that much modern thermodynamic data are referred to the triple point rather than the ice point.

[24] The atmosphere (symbol atm) is defined as 101325 Pa, where Pa = pascal = N·m^{-2} [25.1].

[25] This is the value of the litre recommended by the 11th General Conference of Weights and Measures (CGPM) (1960), defined as the volume of 1 kg of pure water at its maximum density. This definition was abrogated by the 12th CGPM (1964) which recommended that "the name litre should not be employed to give results of high accuracy volume measurements" but declared that the word "may be employed as a special name for the cubic decimeter" [25.1].

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26. Stefan-Boltzmann Constant, σ

The Stefan-Boltzmann constant is of interest over and above its practical use in the field of absolute radiometry because of its relationship with other fundamental constants. The theoretical expression for σ is [0.1]:

$$\sigma = 2\pi^5 k^4/15h^3c^2, \qquad (26.1)$$

which becomes, using eqs (25.5) and (25.1),

$$\sigma = (2\pi^5 p_0{}^4/15T_0{}^4)(V_m{}^4/N_A{}^4h^3c^2). \qquad (26.2)$$

The 31 ppm uncertainty in V_m [eq (25.2)] is by far the largest uncertainty associated with the quantities appearing on the right-hand side of this equation. The implication is that for an experimental determination of σ to carry the same weight in an adjustment as the present best value of V_m, it must have an uncertainty of order 125 ppm. Unfortunately, the present best experimental value of σ, that recently obtained by Blevin and Brown [26.1] at NSL, has an uncertainty of approximately 500 ppm:

$$\sigma = 5.6644(29) \times 10^{-8}\mathrm{W\cdot m^{-2}\cdot K^{-4}}. \qquad (26.3)$$

(We have converted Blevin and Brown's originally quoted 99 percent confidence level uncertainty to a standard deviation. Their paper should also be consulted for a summary of previous determinations of σ.) Clearly, it would not be productive to include σ in our adjustment at this time. Instead, we shall simply calculate a best value of σ from eq (26.2) using the final output values of our adjustment and the Batuecas value for V_m, eq (25.2). Improved measurements of σ could, of course, alter this procedure in future adjustments.

27. Summary of The Less Precise Data

Table 27.1 summarizes the less precise data discussed in sections 12 through 23. (The equation numbers used in the text for these quantities are indicated in the column headed "Eq. No.") The data of table 27.1 will constitute the stochastic input data which will be considered for possible inclusion in our least-squares adjustment. The required auxiliary constants will be taken from table 11.1. The reason $\Omega_{\mathrm{Bi69}}/\Omega$ is not assumed to be an auxiliary constant will be explained in section II.A.29.

We have, of course, already eliminated from table 27.1 some of the experimental results given in sections 12 through 23. Throughout our discussion of the less precise data, we have anticipated this deletion by pointing out when we believed there was sufficient cause to exclude a particular result. In summary, those items which might have been considered for inclusion but which have already been discarded and the reasons for doing so, are:

Equation (12.6), A_{Bi69}/A (VNIIM current balance): The final analysis of the current distribution correction has not yet been completed.

Equation (13.3), F (Bower, iodine coulometer): The experiment was preliminary and systematic effects have not yet been investigated.

Equations (15.1) to (15.6), μ'_p/μ_N (various, low precision): These values have uncertainties much larger than the two sub-ppm determinations.

Equation (16.4), Λ (Tyrén): The uncertainties associated with the Al$K\alpha$ x-ray line preclude the use of this result.

Equation (16.6) and (17.4), Λ and $N_A\Lambda^3$ (Deslattes and Sauder, x-ray-optical interferometer): The experiment was preliminary and several systematic effects have yet to be investigated.

TABLE 27.1. Summary of the stochastic input data to be considered for use in the present work as discussed in sections 12 through 23

Publication date and author	Quantity	Method	Value	Uncertainty (ppm)	Eq. No.
Derived sec. II.A.4 from data of Thompson	$\Omega_{\mathrm{Bi69}}/\Omega$	Calculable capacitor	0.99999946(19)	0.19	(4.4)
1968, Driscoll and Olsen	A_{Bi69}/A	NBS Pellat balance	1.0000018(97)	9.7	(12.1)
1958, Driscoll and Cutkosky	A_{Bi69}/A	NBS current balance	0.9999988(77)	7.7	(12.2)
1965, 1970, Vigoureux	A_{Bi69}/A	NPL current balance	1.0000000(55)	5.5	(12.3)
1960, Craig et al.	F	Silver-perchloric acid coulometer	$9.648672(66) \times 10^4\ A_{\mathrm{Bi69}}\cdot\mathrm{s\cdot mol^{-1}}$	6.8	(13.1)
1968, Marinenko and Taylor	F	Benzoic and oxalic acid coulometers	$9.648695(93) \times 10^4\ A_{\mathrm{Bi69}}\cdot\mathrm{s\cdot mol^{-1}}$	9.6	(13.2)

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TABLE 27.1. Summary of the stochastic input data to be considered for use in the present work as discussed in sections 12 through 23—Continued

Publication date and author	Quantity	Method	Value	Uncertainty (ppm)	Eq. No.
1968, Hara et al.	γ'_p	Low field	$2.6751156(107) \times 10^8 s^{-1} \cdot T_{B169}^{-1}$	4.0	(14.1)
1972, Olsen and Driscoll	γ'_p	Low field	$2.6751370(54) \times 10^8 s^{-1} \cdot T_{B169}^{-1}$	2.0	(14.2)
1965, Vigoureux	γ'_p	Low field	$2.6751187(107) \times 10^8 s^{-1} \cdot T_{B169}^{-1}$	4.0	(14.3)
1971, Malyarevskaya, Studentsov, and Shifrin	γ'_p	Low field	$2.6751100(161) \times 10^8 s^{-1} \cdot T_{B169}^{-1}$	6.0	(14.4)
1966, Yagola, Zingerman, and Sepetyi	γ'_p	High field	$2.675130(20) \times 10^8 A_{B169} \cdot s \cdot kg^{-1}$	7.4	(14.5)
1971, Kibble and Hunt	γ'_p	High field	$2.675075(43) \times 10^8 A_{B169} \cdot s \cdot kg^{-1}$	16	(14.6)
1972, Mamyrin, Aruyev, and Alekseenko	μ'_p/μ_N	Mass spectrometer	2.7927738(12)	0.43	(15.7)
1972, Petley and Morris	μ'_p/μ_N	Omegatron	2.7927748(23)	0.82	(15.8)
1931, Bearden (revised 1964, I. Henins and Bearden)	Λ	Plane ruled grating	1.002027(33)	33	(16.3)
1971, A. Henins	Λ	Plane ruled grating	1.0020655(98)	9.8	(16.5)
1964, Spijkerman and Bearden	Λ	*hc/e*, short wavelength limit	1.002041(33)	33	(16.7)
1964, I. Henins and Bearden	$V_A A^3$	X-ray crystal density (Si)	$6.059768(95) \times 10^{23} mol^{-1}$	16	(17.1)
1965, Bearden	$V_A A^3$	X-ray crystal density (calcite)	$6.05961(17) \times 10^{23} mol^{-1}$	28	(17.2)
1964, Knowles	λ_C	Electron-positron annihilation-Ta	$24.21416(37) \times 10^{-3}$ kxu	15	(18.2)
1971, Van Assche et al.	λ_C	Electron-positron annihilation-Ta	$24.21315(80) \times 10^{-3}$ kxu	33	(18.3)
1971, Wesley and Rich (revised 1972, Granger and Ford)	α^{-1}	Electron anomalous moment, plus theory	137.03563(42)	3.1	(19.8)
1972, Crowe, Williams et al.	μ_μ/μ_p	Muon precession	3.1833467(82)	2.6	(21.1)
1970, Hutchinson et al.	μ_μ/μ_p	Muon precession	3.183356(31)	9.6	(21.2)
1970, DeVoe, Telegdi, et al. (revised 1972, Jarecki and Herman)	μ_μ/μ_p	Muonium Zeeman transitions	3.183350(15)	4.7	(21.5)
Derived sec. II.C.22 from Chicago and Yale data	ν_{Mhfs}	Muonium	4463303.8(1.8) kHz	0.40	(22.2)
1970, Hellwig et al.; 1971, Essen, et al.	α^{-1}	ν_{Hhfs}, hydrogen maser, plus theory	137.03597(22)	1.6	(22.6)
1972, Baird et al.	α^{-1}	Fine-structure splitting ΔE in H, $n = 2$, plus theory	137.03544(54)	3.9	(23.4)
1971, Kaufman, Lamb, et al.	α^{-1}	$(\Delta E - \mathcal{S})$ splitting in H, $n = 2$, plus theory	137.03416(20)	1.5	(23.5)
1971, Shyn et al.	α^{-1}	Same	137.03508(46)	3.3	(23.6)
1970, Cosens and Vorburger	α^{-1}	Same	137.03563(31)	2.3	(23.7)

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Equation (18.1), λ_C (Knowles, H_2O): At best, the experiment was highly preliminary.

Equation (23.8), $\alpha^{-1}(\nu_{01})$(Kponou, Hughes, et al.): The present theory of the helium fine-structure has not been carefully checked.

We note that there are 31 items of stochastic data to be considered.

III. Analysis of Stochastic Input Data

We now turn our attention to the analysis of the stochastic data summarized in table 27.1. The WQED and QED data will first be considered separately, and then together. The tools we use for our investigations include the simple weighted mean of like constants, certain equations which relate the constants of interest to each other, and a large number of least-squares adjustments in which various input items are systematically deleted (a so-called analysis of variance [0.1]). Our statistical tools will be the Birge ratio, R_B, and χ^2. In general, we shall investigate the agreement of like data before investigating the agreement of dissimilar data.

A. The WQED Data

Here we look at the overall consistency of the WQED data, that is, the first 21 items of table 27.1[eqs (4.4) through (18.3)].

28. Inconsistencies Among Data of the Same Kind

Table 28.1 summarizes an analysis of the similar WQED data using their weighted means. Column one gives the quantity in question and column two the items (as indicated by their Eq. Nos.) used in computing these means. The sixth column gives χ^2 and the number of degrees of freedom, ν. (Here, ν equals the number of items minus one.) The last column gives the approximate probability that a value of χ^2 as large or larger than the observed value can occur by chance.[26] It should be noted that a small probability indicates that the data are apparently inconsistent; they differ by relatively large amounts compared with their assigned uncertainties, thereby implying that these uncertainties may have been underestimated. On the other hand, a probability near unity implies that the data are highly consistent; they differ by amounts much smaller than their assigned uncertainties, thereby implying that these uncertainties may be too large. A small value of χ^2, while still equally as improbable as a large one, is not particularly troublesome; we shall therefore take the conservative approach and refrain from reducing any uncertainties. In contrast, inconsistent data are a major concern in any adjustment and we shall seriously consider the possibility of expanding the relevant assigned uncertainties when the circumstances would appear to warrant doing so.

With these remarks in mind and taking the usual 5 percent probability level as the critical "point of concern", we note that none of the data are in such disagreement as to be a major problem. Nevertheless, we are somewhat disturbed at the obvious difference between the NBS low field value of γ'_p, and those obtained in ETL, NPL, and VNIIM which are clearly in excellent agreement among themselves. (The NBS result exceeds the mean of the other three by (7.9 ± 3.2)ppm.) This concern is due to the critical role γ'_p(low) plays in determining a WQED value of α and

[26] These probabilities have been obtained from ref. [14.18], p. 978.

TABLE 28.1. Summary of analysis of WQED data of the same kind

Quantity	Eq. nos. of data used	Weighted mean[a]	Uncertainty (ppm)	Birge ratio, R_B	χ^2/ν, ν = degrees of freedom	Approximate probability
K	(12.1), (12.2), (12.3)	1.0000000(41)	4.1	0.17	0.06//2	0.97
F	(13.1), (13.2)	9.648679(54)	5.6	0.21	0.04//1	0.84
γ'_p(low)	(14.1), (14.2), (14.3), (14.4)	2.6751289(42)	1.6	1.43	6.13//3	0.11
NBS deleted	(14.1), (14.3), (14.4)	2.6751158(68)	2.6	0.32	0.20//2	0.90
γ'_p(high)	(14.5), (14.6)	2.6751205(180)	6.7	1.16	1.35//1	0.24
μ'_p/μ_N	(15.7), (15.8)	2.7927740(11)	0.38	0.39	0.15//1	0.70
Λ	(16.3), (16.5), (16.7)	1.0020609(91)	9.1	0.89	1.59//2	0.45
$N_A\Lambda^3$	(17.1), (17.2)	6.059730(83)	14	0.82	0.67//1	0.41
λ_C	(18.2), (18.3)	24.21398(33)	14	1.15	1.31//1	0.25

[a] The units for F are 10^4 $A_{B169}\cdot s\cdot mol^{-1}$; for γ'_p(low), $10^8 s^{-1}\cdot T_{B169}^{-1}$; for γ'_p(high), 10^8 $A_{B169}\cdot s\cdot kg^{-1}$; for $N_A\Lambda^3$, 10^{23} mol^{-1}; and for λ_C, 10^{-3} kxu.

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the fact that the NBS result has the smallest assigned uncertainty. But we shall postpone deciding if any of the γ_p'(low) uncertainties should be modified until the consistency of all of the WQED stochastic input data is investigated.

29. Inconsistencies Among Data of Different Kinds

We first investigate the overall agreement of the WQED data utilizing equations which relate one constant to another, and then more thoroughly by means of an analysis of variance. On the basis of these investigations, we shall decide how the data may best be handled in order to obtain our recommended set of WQED constants. (The actual values for these constants will be given in part IV.) In those cases where one can conveniently isolate a particular constant, the "one dimensional" approach is especially useful because of its relative simplicity and the ease with which its results may be understood. One of the equations of interest has already been given in section II.B.14:

$$K = [\gamma_p'(\text{low})_{\text{BI69}}/\gamma_p'(\text{high})_{\text{BI69}}]^{1/2}; \qquad (14.12)$$

and may be used to check the compatibility of the measurements of the conversion factor K and the low and high field measurements of γ_p' in the following way: We note that the ETL, NPL, and VNIIM γ_p'(low) values are in excellent agreement (see table 28.1) and yield a weighted mean of $2.6751158(68) \times 10^8$ $\text{s}^{-1}\cdot\text{T}_{\text{BI69}}^{-1}$ (2.6 ppm). When separately combined with the KhGNIIM and NPL high field results using eq (14.12), the result is

KhGNIIM: $K = 0.9999973(39)$ (3.9) ppm, (29.1a)

NPL: $K = 1.0000076(81)$ (8.1) ppm. (29.1b)

If the NBS low field γ_p' result is used in place of this weighted mean, the result is:

KhGNIIM: $K = 1.0000013(38)$ (3.8) ppm, (29.2a)

NPL: $K = 1.0000116(81)$ (8.1) ppm. (29.2b)

The *indirect* values of K given in eqs (29.1) and (29.2) may now be compared with the weighted mean of the three *direct* measurements of K which are in excellent agreement (see table 28.1):

NBS, NPL (direct, wtd. mean):

$$K = 1.0000000(41)\ (4.1)\ \text{ppm}. \qquad (29.3)$$

It may be concluded from the comparison that there are no outstanding discrepancies among these data, although the direct determinations of K are in slightly better agreement with the ETL, NPL, and VNIIM values of γ_p'(low) than with the NBS value. However, it is not statistically significant. (The differences between eqs (29.1a) or (29.1b), and (29.3) are, respectively (-2.7 ± 5.6) ppm, and (7.6 ± 9.1) ppm; between eqs (29.2a) or (29.2b), and (29.3), (1.3 ± 5.6) ppm, and (11.6 ± 9.1) ppm.) Of more importance is the fact that these indirect values of K have accuracies comparable with the directly determined values and will therefore be a significant factor in determining the final value of this quantity in our adjustment.

The second equation we shall use is [0.1]

$$F = \frac{M_p\gamma_p'(\text{low})}{K^2\mu_p'/\mu_N} = \frac{M_p\gamma_p'(\text{high})}{\mu_p'/\mu_N}, \qquad (29.4)$$

where it is understood that F, K, and γ_p' are to be expressed in terms of the same as-maintained electrical units (BI69 units in the present case). Taking M_p as given in table 11.1, K (as needed) equal to the weighted mean of the three highly compatible direct determinations (table 28.1), and μ_p'/μ_N equal to the weighted mean of the two high precision determinations which are also in good agreement (table 28.1), we find from the values of γ_p' indicated (in units of $\text{A}_{\text{BI69}}\cdot\text{s}\cdot\text{mol}^{-1}$)

ETL, NPL, VNIIM: $F_{\text{BI69}} = 96484.04(82)$ (8.5 ppm), (wtd. mean) (29.5a)

NBS: $F_{\text{BI69}} = 96484.80(81)$ (8.4 ppm), (29.5b)

KhGNIIM: $F_{\text{BI69}} = 96484.56(71)$ (7.4 ppm), (29.5c)

NPL(high): $F_{\text{BI69}} = 96482.57(1.55)$ (16 ppm). (29.5d)

These indirect values of the Faraday may be compared with the weighted mean of the two direct NBS Faraday measurements which are in good agreement with each other (see table 28.1):

NBS (direct, wtd. mean):

$$F_{\text{BI69}} = 96486.79(54)\ (5.6\ \text{ppm}). \qquad (29.6)$$

The comparison shows that this direct value exceeds the four indirect values [eq (29.5)] by, respectively, (29 ± 10) ppm, (21 ± 10) ppm, (23 ± 9.3) ppm, and (44 ± 17) ppm. These differences, taken together, imply that the directly determined values of F are in disagreement with the available K, γ_p', and μ_p'/μ_N data. Indeed, if all of the latter data are combined in a least-squares adjustment in order to determine a single best indirect value of F,[27] we find

indirect: (least-squares) $F_{\text{BI69}} = 96484.33(50)$ (5.2 ppm). (29.7)

[27] In this adjustment, the input items were eqs (12.1) (12.2), (12.3), (14.1) through (14.6), and (15.7) and (15.8). χ^2 is 7.77 (8 degrees of freedom), and $R_B = 0.99$.

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The direct value, eq (29.6), exceeds this result by (26 ± 7.6) ppm. It must therefore be concluded that the Faraday determinations are sufficiently discrepant that serious consideration must be given to the possibility of excluding them from our adjustment.

The present Faraday situation is, of course, reminiscent of that faced by Taylor et al. in their 1969 adjustment in which the indirect values of F implied by the so called "low values" of μ_p'/μ_N were in good agreement with the NBS Craig et al. Faraday measurement, but the indirect values of F implied by the "high values" of μ_p'/μ_N were in disagreement with it. Primarily on this basis, Taylor et al. discarded the two high values of μ_p'/μ_N, retaining the low values and F. In the present case, the very high precision of the two μ_p'/μ_N measurements and their excellent agreement point the finger of suspicion unequivocally at the Faraday measurements.

The final quantity we consider by means of simple equations is Λ. It may be shown [0.1] that

$$\Lambda = \frac{10^{10}\alpha^2}{2R_\infty\lambda_C}, \tag{29.8}$$

where λ_C is to be expressed in kxu and R_∞ in m^{-1}. Since the uncertainties of the two experimental values of λ_C, eqs (18.2) and (18.3), are of order 15 and 30 ppm, respectively, the value of α^{-1} used in this equation is not critical. Anticipating the result of our adjustment, we shall assume $\alpha^{-1} = 137.0360 \pm 1$ ppm. Taking R_∞ as given in table 11.1 then yields

Knowles: $\Lambda = 1.002021(15)$ (15 ppm), (29.9a)

Van Assche et al.:
$\Lambda = 1.002063(33)$ (33 ppm). (29.9b)

These indirect values may be compared with the weighted mean of the three more or less direct determinations which are in good agreement (see table 28.1):

direct: $\Lambda = 1.0020609(91)$ (9.1 ppm).
(wtd. mean) (29.10)

Although eqs (29.9b) and (29.10) are obviously highly consistent, a comparison of eqs (29.9a) and (29.10) shows that the latter exceeds the former by (40 ± 18) ppm. Thus, the x-ray data may well provide us with another discrepancy of serious proportions.

We now turn our attention to a least-squares analysis of the WQED data. We take as the unknowns or "adjustable constants" α^{-1}, K, N_A, Ω_{B169}/Ω, and Λ. All of the WQED stochastic data may be individually expressed in terms of this set of variables with the aid of appropriate auxiliary constants (table 11.1). Our

TABLE 29.1. Observational equations used in the present adjustment

Quantity	Equation[a]
$\Omega_{B169}/\Omega \equiv \bar{R}$	$\bar{R} = \{\bar{R}\}$
$A_{B169}/A \equiv K$	$K = \{K\}$
F	$\alpha N_A \bar{R} = (\mu_0/4)c(2e/h)_{B169}\{F_{B169}\}$
γ_p'(low field)	$\alpha^2\bar{R}^{-1} = \dfrac{4R_\infty}{c(\mu_p'/\mu_B)(2e/h)_{B169}}\{\gamma_p'(\text{low})_{B169}\}$
γ_p'(high field)	$\alpha^2K^{-2}\bar{R}^{-1} = \dfrac{4R_\infty}{c(\mu_p'/\mu_B)(2e/h)_{B169}}\{\gamma_p'(\text{high})_{B169}\}$
μ_p'/μ_N	$\alpha K^{-2}N_A^{-1}\bar{R}^{-2} = \dfrac{16R_\infty}{M_p\mu_0(\mu_p'/\mu_B)c^2(2e/h)^2_{B169}}\{\mu_p'/\mu_N\}$
Λ	$\Lambda = \{\Lambda\}$
$N_A\Lambda^3$	$N_A\Lambda^3 = \{N_A\Lambda^3\}$
λ_C	$\alpha^2\Lambda^{-1} = 2 \times 10^{-10} R_\infty\{\lambda_C\}$
α^{-1}	$\alpha^{-1} = \{\alpha^{-1}\}$
$\mu_\mu/\mu_p \equiv \mu$	$\mu = \{\mu\}$
ν_{Mhfs}	$\alpha^2\mu = \dfrac{3(1 + m_e/m_\mu)^3\,[1.00019721(201)]}{16R_\infty c(\mu_e/\mu_B)(\mu_p/\mu_B)}\{\nu_{Mhfs}\}$

[a] The braces indicate numerical value only.

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variables differ from those used by Taylor et al. [0.1] (namely, α^{-1}, e, K, N_A, and Λ), for two reasons. First, since $(2e/h)_{BI69}$ is now an auxiliary constant, the elementary charge, e, can be expressed in terms of the new variables and $(2e/h)_{BI69}$:

$$e = \frac{\alpha K(\Omega_{BI69}/\Omega)}{c(\mu_0/4)(2e/h)_{BI69}}. \qquad (29.11)$$

Thus it is no longer stochastically independent of the other adjustable variables and may be eliminated from the least-squares solution. Second, the uncertainty in the two best experimental determinations of μ_p'/μ_N is at the parts in 10^7 level, quite comparable with the $2/10^7$ uncertainty in Ω_{BI69}/Ω which enters the observational equation for μ_p'/μ_N to the second power. Since Ω_{BI69}/Ω also enters the observational equations for γ_p' (low) and γ_p'(high), and must be used to obtain a best value of V_{BI69}/V from K, it is necessary to consider it an adjustable quantity, thereby properly taking into account its correlations with the other data.

The various observational equations for the quantities of interest are summarized in table 29.1. (Note that we have used the symbols $\bar{R} \equiv \Omega_{BI69}/\Omega$ and $\mu \equiv \mu_\mu/\mu_p$.) The last three equations are for the QED data and do not concern us here; they will be discussed in sections III.C.30 and 31. The braces indicate numerical value only, and the subscript BI69 means, of course, that the quantity in question is to be expressed in BIPM 1 January 1969 units as discussed in sections

TABLE 29.2. Summary of least-squares adjustments involving the WQED data only

Adjust. No.	Eq. Nos. of items deleted	Birge ratio, R_B	χ^2/ν, ν = degrees of freedom	Adjusted values, X_i[a] Δ_i				
				α^{-1}	e	K	N_A	Λ
1	None	1.50	36.02//16	0.40 ± 0.78	−3.7 ± 2.4	−5.5 ± 2.0	16.4 ± 3.8	−4.6 ± 4.0
2	(18.2)	1.27	24.20//15	0.65 ± 0.78	−3.7 ± 2.4	−5.2 ± 2.0	15.7 ± 3.8	−0.8 ± 4.2
3	(18.2), (13.1), (13.2)	1.05	14.44//13	0.85 ± 0.78	1.3 ± 2.8	0.0 ± 2.6	5.0 ± 5.1	2.0 ± 4.3
4	(18.2), (15.7), (15.8)	0.94	11.48//13	0.87 ± 0.78	2.2 ± 2.9	0.9 ± 2.6	29.9 ± 5.5	−4.6 ± 4.3
5	(18.2), (12.1), (12.2), (12.3)	1.35	21.96//12	0.84 ± 0.79	−5.5 ± 2.7	−6.9 ± 2.3	18.8 ± 4.3	−1.6 ± 4.2
6	(18.2), (14.5), (14.6)	1.13	16.52//13	0.53 ± 0.78	−6.7 ± 2.6	−8.3 ± 2.3	22.0 ± 4.6	−2.5 ± 4.2
7	(18.2), (12.1), (12.2), (12.3), (14.5), (14.6)	1.01	10.13//10	0.84 ± 0.79	−11.1 ± 3.2	−12.5 ± 2.8	29.9 ± 5.5	−4.6 ± 4.3
8	(18.2), (14.2)	1.19	19.81//14	2.6 ± 1.2	−7.2 ± 2.9	−6.8 ± 2.1	16.8 ± 3.8	−1.2 ± 4.2
9	(18.2), (14.1), (14.3), (14.4)	1.20	17.32//12	−0.88 ± 0.98	−1.0 ± 2.6	−4.0 ± 2.0	14.8 ± 3.8	−0.6 ± 4.2
10	(18.2), (13.1), (13.2), (12.1), (12.2), (12.3)	1.20	14.38//10	0.85 ± 0.79	1.3 ± 3.6	0.0 ± 3.4	5.0 ± 6.6	2.0 ± 4.4
11	(18.2), (13.1), (13.2), (14.5), (14.6)	1.07	12.56//11	0.76 ± 0.79	−0.7 ± 4.0	−2.1 ± 3.9	9.3 ± 7.9	0.8 ± 4.5
12	(18.2), (13.1), (13.2), (14.2)	0.84	8.55//12	3.2 ± 1.2	−2.4 ± 3.2	−1.4 ± 2.7	5.5 ± 5.1	1.8 ± 4.3
13	(18.2), (13.1), (13.2), (14.3), (14.4)	0.92	8.51//10	−0.58 ± 0.99	3.6 ± 3.0	0.8 ± 2.6	4.7 ± 5.1	2.1 ± 4.3
14	(18.2), (13.1), (13.2), (16.3), (16.5), (16.7), (18.3)	0.97	8.44//9	0.86 ± 0.79	2.3 ± 2.9	0.9 ± 2.6	3.1 ± 5.2	7.0 ± 4.9
15	(18.2), (13.1), (13.2), (17.1), (17.2)	0.92	9.35//11	0.86 ± 0.78	2.3 ± 2.9	0.9 ± 2.6	3.1 ± 5.2	−14.1 ± 8.7

[a] The numbers given are the differences in ppm between the adjusted values X_i and the following exact fiducial values X_{oi}: $\alpha_o^{-1} = 137.0360$; $e_o = 1.602185 \times 10^{-19}$ C; $K_o = 1.000000$; $N_{Ao} = 6.022020 \times 10^{23}$ mol^{-1}; $\Lambda_o = 1.002075$. That is, $X_i = X_{oi}[1 + \Delta_i \times 10^{-6}]$.

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II.A.2 and 4; Λ, $N_A\Lambda^3$, and λ_C are to be expressed in kxu.

Table 29.2 summarizes a series of least-squares adjustments involving the 21 items of WQED stochastic data with α^{-1}, K, N_A, $\bar{R}$ and Λ as the five unknowns. The purpose of the table is to give some indication of the compatibility of the data and the variability of the adjusted values of α^{-1}, e, K, N_A, and Λ for different selections of input data. $\bar{R}$ has not been shown because it varies very little; e is calculated from eq (29.11) and its uncertainty obtained from the error matrix in the usual way. (A discussion of the error matrix and its use is given briefly in section IV.B.33. For more details see Taylor et al. [0.1] or Cohen et al. [29.1, 29.2].) The table is more or less self explanatory, and we therefore limit ourselves to general remarks intended as a guide to the various adjustments.

Adjustment No. 1, which involves all of the WQED data (the first 21 items of table 27.1), has a Birge ratio (or χ^2) sufficiently large that it must be concluded that the data are incompatible. This is due in part to the Knowles λ_C result, eq (18.2), the largest single contributor to χ^2; its normalized residual is 3.30.[28] Deleting this one item reduces χ^2 by 11.82 (adjustment No. 2). Indeed, the Knowles λ_C result is so clearly discrepant with the other data that it must be excluded from further consideration. (Taylor et al. came to this same conclusion [0.1].)

[28] The normalized residual for a particular stochastic input datum, r_i, is the deviation of the datum from its least-squares adjusted value divided by (or normalized to) the *a priori* uncertainty of the datum. Since χ^2 is equal to $\sum_{i=1}^{N} r_i^2$ and the expectation value of χ^2 is $N-J=\nu$=degrees of freedom, where N is the total number of input items and J is the number of unknowns, the average contribution of each r_i^2 to χ^2 is $(N-J)/N$. Thus, if a particular input datum has a normalized residual much in excess of unity, the datum is incompatible with the remaining data. (See ref. [0.1] for a further discussion.)

The effect of deleting the suspect Faraday data is shown in adjustment No. 3. The Birge ratio becomes very nearly equal to unity; and significant shifts occur in e, K, N_A, and Λ, especially in N_A. The latter arises because $N_A=F/e$, and e depends only weakly on F through α and K [see eq (29.11)]. The close relationship between F and μ_p'/μ_N indicated by eq (29.4) is shown by the results of adjustment No. 4 in which the two high precision measurements of μ_p'/μ_N are deleted; R_B is less than 1, and N_A changes by some 25 ppm (~5 standard deviations). Thus, as previously noted, the Faraday discrepancy is mainly a discrepancy between the values of F and μ_p'/μ_N. Since according to eq (29.4), F and μ_p'/μ_N are coupled to each other through K and γ_p', it is of interest to see what occurs when these quantities are deleted and F and μ_p'/μ_N are retained. This is shown in adjustments No. 5 through 9 in which the three direct measurements of K are deleted (No. 5); the two high field γ_p' values are excluded (No. 6); both the K and γ_p' (high) measurements are deleted (No. 7); the NBS value of γ_p' (low) is deleted (No. 8); and the ETL, NPL, and VNIIM values of γ_p' (low) are excluded (No. 9). A comparison of adjustments No. 8 and No. 9 is of particular interest because it clearly shows the critical dependence of α on the different values of γ_p' (low). (See also adjustments No. 12 and 13.) This will be discussed in greater detail in section III.C.31.

The last six adjustments give an indication of how the data interact when the Faraday measurements are

TABLE 29.3. Summary of least-squares adjustments involving the WQED data with selected expanded uncertainties

Adjust. No.	Eq. Nos. of items deleted	Birge ratio, R_B	$\chi^2//\nu$, ν = degrees of freedom	Adjusted values, X_i[a] Δ_i				
				α^{-1}	e	K	N_A	Λ
γ_p'(low) uncertainties expanded by 1.43								
16	None	1.43	32.62//16	0.0 ± 1.1	−2.9 ± 2.7	−5.1 ± 2.1	16.1 ± 3.8	−4.5 ± 4.0
17	(18.2)	1.18	21.03//15	0.5 ± 1.1	−3.4 ± 2.7	−5.1 ± 2.1	15.6 ± 3.8	−0.8 ± 4.2
18	(18.2), (13.1), (13.2)	0.93	11.31//13	0.9 ± 1.1	1.3 ± 3.1	0.0 ± 2.6	5.0 ± 5.1	2.0 ± 4.3
γ_p'(low) and x-ray data uncertainties expanded by 1.43 and 1.28, respectively								
19	None	1.28	26.18//16	0.2 ± 1.1	−2.9 ± 2.7	−5.0 ± 2.1	15.6 ± 3.8	−4.4 ± 5.1
20	(18.2)	1.13	19.02//15	0.5 ± 1.1	−3.2 ± 2.7	−4.9 ± 2.1	15.3 ± 3.8	−0.7 ± 5.3
21	(18.2), (13.1), (13.2)	0.82	8.75//13	0.9 ± 1.1	1.6 ± 3.1	0.3 ± 2.6	4.3 ± 5.2	2.1 ± 5.3

[a] See footnote, table 29.2.

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deleted along with other quantities. In adjustment No. 10, the three values of K are deleted; in No. 11, the two values of γ_p'(high); in No. 12, the NBS result for γ_p' (low); in No. 13, the other three values of γ_p'(low); in No. 14, the more or less direct values of Λ including Van Assche et al.'s λ_C result; and in No. 15, the two $N_A\Lambda^3$ results. The latter two adjustments deserve special attention because they clearly show the difference between the value of Λ implied by the three direct measurements and the Van Assche measurement of λ_C, and that implied by the two $N_A\Lambda^3$ determinations; there is a 21 ppm difference between the two. With regard to the x-ray data, we note that they play a very small role in determining the values of any of the constants except Λ (compare adjustment No. 3 with either Nos. 14 or 15).

It was pointed out in the previous section that the overall agreement of the low field γ_p' measurements was less than satisfactory in view of this quantity's critical role in any adjustment. We have therefore investigated the effect of expanding the *a priori* uncertainties of the four low field values by the Birge ratio associated with their weighted mean, namely, by the multiplicative factor 1.43 (see table 28.1, line 3). In other words, we reassign uncertainties to the γ_p'(low) data which are based on external consistency (σ_E) rather than use their *a priori* assigned uncertainties. Table 29.3 (first portion) gives the results of three adjustments in which the uncertainties of the γ_p'(low) determinations have been increased by the above factor. In comparing these adjustments to their counterparts in table 29.2 (i.e., Nos. 16, 17, and 18 with Nos. 1, 2, and 3) we see that R_B has decreased, that the values of the adjusted constants have changed by relatively small amounts, and that their uncertainties have increased. We believe these increased uncertainties are more realistic in view of the variation in the adjusted constants with the choice of input data even after the highly discrepant data are deleted (Knowles' λ_C value and the two values of F).

In this same vein, we note that in adjustment No. 18, the contribution of the six items of x-ray data to the overall χ^2 is 6.54. Assuming that these six items of data determine the two quantities N_A and Λ, the implied Birge ratio is $[6.54/(6-2)]^{1/2} = 1.28$. (Adjustment No. 3, in which the *a priori* uncertainties assigned the γ_p'(low) data are not expanded, yields the same factor since the x-ray data are only weakly coupled to the other WQED data.) The effect of expanding (multiplicatively) all of the x-ray data uncertainties by this amount is shown in the bottom portion of table 29.3. Clearly, the remarks made above concerning the effect of expanding just the γ_p'(low) uncertainties apply here as well. In particular, in comparing adjustment No. 21 with No. 3, it may be seen that any changes in the numerical values of the adjusted constants are only small fractions of their uncertainties.

In conclusion, we believe that adjustment No. 21 represents the most reasonable way of handling the WQED data. The increased uncertainties in the adjusted constants resulting from expanding the *a priori* uncertainties of the γ_p'(low) and x-ray data more nearly reflect the overall variability of the values of the adjusted constants with the particular selection of input data. The normalized residuals of the 18 items of input data are less than unity, the only exceptions being the NBS γ_p'(low) result, the NPL γ_p'(high) result, and the 1931 Bearden-Henins value of Λ [eqs (14.2), (14.6), (16.3)]. For these, r_i = 1.09, 1.20, and 1.17, respectively, which are not unreasonable. The overall value of χ^2 = 8.75 for the adjustment (13 degrees of freedom) is quite satisfactory (R_B=0.82). We do not believe that our approach is too conservative as one might assume at first glance from this value of χ^2 because two groups of data are in such abnormally good agreement among themselves that their own internal χ^2 contributes very little to the overall χ^2 (K and μ_p'/μ_N; see table 28.1). Indeed, if adjustments are carried out in which we use eight input equations, each representing the weighted mean of each of the different kinds of WQED data in table 27.1 (excluding Knowles' λ_C result and the two Faraday measurements), then we find χ^2 = 4.50 for the adjustment corresponding to No. 3 (R_B=1.22); χ^2=4.50 for the adjustment corresponding to No. 18 (R_B=1.22); and χ^2=2.82 for the adjustment corresponding to No. 21 (R_B=0.97). In each, the number of degrees of freedom is 8 − 5 = 3. Since χ^2 for such adjustments depends *only* on the compatibility of *dissimilar* kinds of data rather than on the compatibility of *both similar and dissimilar* data, the 2.82 value for χ^2 for 3 degrees of freedom gives perhaps a clearer picture of the general agreement of the data used in our "best" WQED adjustment, No. 21.[29] (The probability for ν=3 that a value of χ^2 as large or larger than 2.82 can occur by chance is 0.42.)

B. The QED Data

We shall now consider the overall consistency of the QED data, that is, the last 10 items of table 27.1 [eqs (19.8) through (23.7)]. These 10 data involve the two quantities α^{-1} and μ_μ/μ_p. There are three direct determinations of μ_μ/μ_p, six determinations of α^{-1} (anomalous electron magnetic moment, fine-structure measurements, and hydrogen hyperfine splitting), and one measurement which combines the two (muonium hyperfine splitting).

30. Inconsistencies Among the QED Data

Table 30.1, which is the QED counterpart of table 28.1, summarizes an analysis of the similar QED data.

[29] One may distinguish between an "intra" χ^2 component arising from the variability of like items of input data: and an "inter" χ^2 component arising from the consistency of dissimilar items of input data. In table 28.1, χ^2 is solely of the "intra" variety. For condensed adjustments such as are being discussed here, χ^2 is solely of the "inter" variety. χ^2 for adjustments such as Nos. 1 through 21 arises from both intra and inter data variability and consistency.

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TABLE 30.1. Summary of analysis of QED data of the same kind

Quantity	Eq. Nos. of data used	Weighted mean	Uncertainty (ppm)	Birge ratio, R_B	χ^2/ν	Approximate chance probability
μ_μ/μ_p	(21.1), (21.2), (21.5)	3.1833479(70)	2.2	0.24	0.11//2	0.95
μ_μ/μ_p	(21.1), (21.2), (21.5), (30.1)	3.1833434(60)	1.9	0.75	1.70//3	0.64
α^{-1}(fs)[a]	(23.4), (23.5), (23.6), (23.7)	137.03472(15)	1.1	2.50	18.70//3	0.0003
Kaufman, Lamb, et al. deleted	(23.4), (23.6), (23.7)	137.03545(23)	1.7	0.71	1.00//2	0.61
α^{-1} (hfs)[b]	(22.4), (22.6)	137.03617(15)	1.1	1.26	1.59//1	0.21
α^{-1} (all)[c]	(19.8), (22.4), (22.6), (23.4), (23.5), (23.6), (23.7)	137.03546(10)	0.75	3.34	66.93//6	2×10^{-12}
Kaufman, Lamb, et al. deleted	(19.8), (22.4), (22.6), (23.4), (23.6), (23.7)	137.03593(12)	0.88	1.40	9.83//5	0.08
Kaufman, Lamb, et al. and α^{-1} (Mhfs) deleted	(19.8), (22.6), (23.4), (23.6), (23.7)	137.03571(15)	1.1	0.95	3.62//4	0.46
Kaufman, Lamb, et al., α^{-1} (Mhfs), and Shyn et al. deleted	(19.8), (22.6), (23.4), (23.7)	137.03579(16)	1.2	0.70	1.48//3	0.69
All four fine-structure measurements deleted	(19.8), (22.4), (22.6)	137.03611(14)	1.0	1.24	3.05//2	0.22
Kaufman, Lamb, et al. and α^{-1} (hfs) deleted	(19.8), (23.4), (23.6), (23.7)	137.03549(20)	1.5	0.61	1.13//3	0.77

[a] fs = the fine structure measurements in hydrogen, ΔE_H and $(\Delta E - \mathcal{S})_H$ (i.e., the values of α^{-1} derived from the determinations of Baird et al.; Kaufman, Lamb, et al.; Shyn et al.; and Cosens and Vorburger).

[b] hfs = the hyperfine splitting measurements in hydrogen and muonium [i.e., α^{-1} (Hhfs) and α^{-1} (Mhfs)].

[c] all = fs plus hfs, plus the measurement of the electron anomalous moment, a_e [i.e., as in a and b plus α^{-1} (a_e)].

The first entry in table 30.1, which compares the three direct measurements of the ratio of the muon to proton magnetic moment, shows the excellent consistency of these determinations. An independent value of this ratio may also be obtained from the ratio $\nu_{\text{Mhfs}}/\nu_{\text{Hhfs}}$ [see eq (22.7)]. Taking $\delta_N^{(2)} = (0 \pm 3)$ ppm as adopted in section II.C.20, we find

$$\text{hfs: } \mu_\mu/\mu_p = 3.1833303(120)\ (3.8\ \text{ppm}). \qquad (30.1)$$

The weighted mean of the three direct values exceeds this value by (5.5 ± 4.4) ppm. (This 5.5 ppm is, of course, the same difference which in eq (22.8) was ascribed tentatively, and alternatively, to proton polarizability). Since the difference is only 1.2 times the standard deviation of the difference, when the hfs value of μ_μ/μ_p is combined with the three direct values (second line of table 30.1), the agreement is quite reasonable.

We now look at the consistency of the hydrogen fine-structure values of α (line 3) and find a serious discrepancy which is removed if eq (23.5) (Kaufman, Lamb, et al.) is omitted (line 4). The discrepant nature of this measurement was discussed to some extent in section II.C.23. Here we see quantitatively that its disagreement with the other fine-structure determinations is so severe that it may have to be excluded from our final adjustment.

When we reverse the argument which led to eq (30.1) and use the muonium hfs measurement and the weighted mean of the three μ_μ/μ_p values to determine α^{-1} (Mhfs) [eq (22.4)], and compute its weighted mean with α^{-1} (Hhfs) [eq (22.6)], line 5 results. The agreement between these two hfs α values is clearly not

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unreasonable. When *all* of the α values implied by the QED data are combined (except that from helium fine-structure—see table 23.1), we obtain the results shown in line 6. The gross inconsistency of the data, as one might expect, is due to the Kaufman, Lamb et al. result and is reduced many orders of magnitude when this determination is deleted (line 7). Although the overall agreement is now statistically acceptable, it is less than satisfactory in view of the critical role the fine-structure constant plays in any adjustment. (The effect of successively deleting the next two most discrepant items, i.e., α^{-1} (Mhfs) and the value of α implied by the Shyn et al. determination of $(\Delta E - \mathcal{S})_H$, is shown in lines 8 and 9 of the table.) Thus, as with the γ'_p (low) data considered previously, we shall at a later point consider the possibility of modifying the uncertainties assigned these data.

It is important to note that the particular method we have used to include the muonium hfs in the analysis of table 30.1 does not alter our conclusions in any way. This may be seen in table 30.2 which summarizes the results of nine least-squares adjustments involving all of the QED data. In these adjustments, α^{-1} and μ_μ/μ_p are taken as the unknown or adjustable constants. (The observational equations used are the last three of table 29.1; a more detailed discussion will be given in the next section.) The very discrepant nature of the Kaufman, Lamb, et al. result is shown by the significant decrease in R_B on going from adjustment No. 1, to adjustment No. 2 in which it has been deleted. Excluding the Shyn et al. result as well reduces R_B still further (no. 3). Deleting the muonium hfs (ν_{Mhfs}), in place of the Shyn et al. result leads to an even lower Birge ratio (No. 4). (Note that when ν_{Mhfs} is deleted, μ_μ/μ_p is simply the weighted mean of the three direct measurements.) The effect of excluding all four hydrogen fine-structure α values is shown in adjustment No. 6 which may be compared with adjustment No. 9 in which ν_{Mhfs} and α^{-1} (Hhfs) are deleted instead. (The obviously discrepant Kaufman, Lamb, et al. determination has also been excluded). Adjustments Nos. 7 and 8 are of special interest because they give indirect values of μ_μ/μ_p, that is, in each the three direct measurements of this quantity have been deleted. (These adjustments correspond to lines 8 and 9, respectively, of table 30.1). In summary, the table clearly shows the wide variation in both α^{-1} and μ_μ/μ_p which can result if various input data are excluded.

C. The WQED and QED Data Together

Here we investigate the overall agreement of all of the stochastic input data listed in table 27.1. On the basis of this analysis, we shall decide how the data may best be handled in order to obtain our final recommended set of constants. The actual values for these constants will be given in part IV.

31. Overall Data Compatibility

Some insight into the overall compatibility of the stochastic data may be obtained by comparing the QED values of the fine-structure constant, α, with those values of α which may be readily derived from

TABLE 30.2. Summary of least-squares adjustments of the QED data only

Adjustment No.[a]	Eq. Nos. of data deleted	Birge ratio, R_B	χ^2/ν, ν=degrees of freedom	Adjusted values, X_i[b] Δ_i	
				α^{-1}	μ_μ/μ_p
1	None	2.90	67.04//8	−3.93 ± 0.75	−7.6 ± 1.7
2	(23.5)	1.19	9.94//7	−0.50 ± 0.88	−3.9 ± 1.8
3	(23.5), (23.6)	1.02	6.22//6	−0.04 ± 0.91	−3.4 ± 1.8
4	(23.5), (22.2)	0.79	3.74//6	−2.1 ± 1.1	−0.7 ± 2.2
5	(23.5), (23.6), (22.2)	0.56	1.59//5	−1.5 ± 1.2	−0.7 ± 2.2
6	(23.4), (23.5), (23.6), (23.7)	0.89	3.17//4	0.8 ± 1.0	−2.5 ± 1.9
7	(23.5), (21.1), (21.2), (21.5)	0.95	3.62//4	−2.1 ± 1.1	−9.9 ± 3.0
8	(23.5), (23.6), (21.1), (21.2), (21.5)	0.70	1.48//3	1.5 ± 1.2	−8.8 ± 3.1
9	(23.5), (22.4), (22.6)	0.50	1.24//5	−3.7 ± 1.5	−0.7 ± 2.2

[a] Adjustment No. 1 includes the last ten items listed in table 27.1, that is, all of the QED data we are considering in the present work. α^{-1} and μ_μ/μ_p are taken as the adjustable constants.

[b] The numbers given are the differences in ppm between the adjusted values, X_i, and the following exact fiducial values, X_{oi}: $\alpha_o^{-1} = 137.0360$; $(\mu_\mu/\mu_p)_o = 3.183350$. That is, $X_i = X_{oi}[1 + \Delta_i \times 10^{-6}]$.

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the WQED data. As has now become well known, WQED α values may be obtained from the following relation [0.1]:

$$\alpha^{-1} = \left[\frac{1}{4R_\infty}\frac{c}{(\Omega_{\mathrm{Bl69}}/\Omega)}\frac{\mu_p'}{\mu_B}\frac{(2e/h)_{\mathrm{Bl69}}}{\gamma_p'(\mathrm{low})_{\mathrm{Bl69}}}\right]^{1/2}. \quad (31.1)$$

Using the values given in table 11.1 for the more precisely known constants in this equation, we find, respectively, for the NBS determination of γ_p'(low), and the weighted mean of the highly compatible ETL, NPL, and VNIIM γ_p'(low) measurements,

NBS: $\alpha^{-1} = 137.03591(14)$ (1.0 ppm), (31.2a)

ETL, NPL, VNIIM: $\alpha^{-1} = 137.03645(18)$ (1.3 ppm). (wtd. mean) (31.2b)

(The (4.0 ± 1.6) ppm difference between the two values of α is, of course, due to the (7.9 ± 3.2) ppm difference between the two values of γ_p'(low) previously noted in section III.A.28.) From a comparison of these WQED α values with those implied by the QED data as given in table 23.1, it may be concluded that eqs (31.2a) and (31.2b) are in better agreement with the hyperfine splitting α values than with the fine-structure values; and that these indirect α values have uncertainties comparable with those assigned the QED values and will therefore play an important role in determining the final value of α in our adjustment.

The two high field determinations of γ_p' and the three direct measurements of K may be compared with the QED data by noting that in eq (31.1), $\gamma_p'(\mathrm{low})_{\mathrm{Bl69}}$ may be replaced by $K^2\gamma_p'(\mathrm{high})_{\mathrm{Bl69}}$ [see eq. (14.12)]. Taking K equal to the weighted mean of the three highly compatible direct determinations (table 28.1), we find

KhGNIIM: $\alpha^{-1} = 137.03608(75)$ (5.5 ppm), (31.3a)

NPL: $\alpha^{-1} = 137.03749(123)$ (9.0 ppm). (31.3b)

Clearly, these indirect values of α are quite consistent with the QED values, table 23.1 (differences of less than two standard deviations); but of course, they are of relatively low accuracy.

Finally, we compare the experimental values of the Faraday constant to the QED data using the equation

$$\alpha^{-1} = \left[\frac{M_p}{4R_\infty}\frac{c}{(\Omega_{\mathrm{Bl69}}/\Omega)}\frac{\mu_p'}{\mu_B}\frac{(2e/h)_{\mathrm{Bl69}}}{K^2F_{\mathrm{Bl69}}(\mu_p'/\mu_N)}\right]^{1/2}, \quad (31.3)$$

which may be obtained by combining eqs (29.4) and (31.1). Taking K^2 equal to the weighted mean of the three direct determinations as above, and μ_p'/μ_N equal to the weighted mean of the two high precision direct determinations which are in good agreement with each other, we find for the weighted mean of the two NBS Faraday measurements (also in good agreement with each other),

NBS: (wtd. mean) $\alpha^{-1} = 137.03449(67)$ (4.9 ppm). (31.4)

This indirect α value is in rather poor agreement with the more accurate QED α values of table 23.1 (differences of two or more standard deviations), except for the Kaufman, Lamb, et al. result which was previously seen to be highly discrepant. Thus, the two Faraday determinations have little support among the QED data and will no doubt have to be discarded as was done when the final data were chosen for our WQED set of "best values" (see sec.III.A.29). We also note that since the WQED data couple to the QED data mainly through the WQED indirect values of α, the above analysis gives a fairly clear picture of the overall agreement of the two groups of data. In summary, apart from the values of α implied by the direct determinations of F, they appear to be relatively compatible.

We now turn our attention to a least-squares analysis of the data. The unknown or adjustable constants may be taken to be the same as those used for the similar analysis of the WQED data with the addition of $\mu \equiv \mu_\mu/\mu_p$. Thus, the six adjustable constants are α^{-1}, K, N_A, $\bar{R}$, Λ, and μ. We include μ as a sixth variable in order to obtain a best value for this quantity from our adjustment since, as may be seen from eq (30.1), the indirect value of μ has an accuracy comparable to the direct values, eqs (21.1), (21.2), and (21.5). Therefore, in the least-squares analysis we use the measurement of muonium hfs as a determination not of α, but of $\alpha^2\mu$ as indicated in eq (20.7b) and more explicitly in table 29.1. It should be remembered that the 2 ppm uncertainty in the numerical factor in the equation for $\alpha^2\mu$ in that table represents our own estimation of the uncertainty in the theory of muonium hyperfine structure and is not determined by the uncertainty in the actual evaluation of that factor from existing theory. Although the numerical value of the factor does depend on α and m_e/m_μ, an accuracy of only 50 ppm in α or m_e/m_μ is necessary to evaluate it with an accuracy of 0.01 ppm if the theory were exact.

Table 31.1, which is the all-data counterpart of table 29.2, summarizes a series of least-squares adjustments involving the entire 31 items of stochastic data listed in table 27.1, with α^{-1}, K, N_A, $\bar{R}$, Λ, and μ as the six unknowns. The purpose of the table is to give some indication of the compatibility of all of the available input data and the variability of the adjusted values of α^{-1}, e, K, N_A, and μ for different selections of data. We omit $\bar{R}$ from this tabulation as we did in table 29.2, and for the same reason. We omit Λ in the interest of brevity and because the major variability in Λ is determined by the WQED data and this has

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TABLE 31.1. Summary of least-squares adjustments involving all of the stochastic data

Adjust. No.	Eq. Nos. of items deleted	Birge ratio, R_B	χ^2/ν, ν=degrees of freedom	Adjusted values, X_i[a] Δ_i				
				α^{-1}	e	K	N_A	μ_μ/μ_p
22	None	2.18	119.05//25	−1.83±0.54	0.4± 2.1	−3.7± 1.9	15.0 ± 3.8	−5.3 ± 1.6
23	(23.5)	1.39	46.55//24	0.01± 0.58	−2.9± 2.2	−5.1± 1.9	16.1 ± 3.8	−3.4 ± 1.6
24	(23.5), (18.2)	1.24	35.09//23	0.14± 0.58	2.8± 2.2	−4.8± 1.9	15.4 ± 3.8	−3.2 ± 1.6
25	(23.5), (18.2), (13.1), (13.2)	1.11	25.68//21	0.25± 0.59	2.3± 2.7	0.3± 2.6	4.9 ± 5.1	−3.1 ± 1.6
26	(23.5), (18.2), (15.7), (15.8)	1.04	22.77//21	0.26± 0.59	3.2± 2.8	1.3± 2.6	29.3 ± 5.5	−3.1 ± 1.6
27	No. 25 + (23.6)	1.03	21.20//20	0.47± 0.60	1.9± 2.7	0.2± 2.6	4.9 ± 5.1	−2.9 ± 1.6
28	No. 25 + (22.2)	1.07	22.98//20	−0.16± 0.64	3.0± 2.8	0.6± 2.6	4.8 ± 5.1	−0.7 ± 2.2
29	No. 25 + (23.4), (23.6), (23.7)	0.99	17.60//18	0.82± 0.62	1.4± 2.7	0.0± 2.6	5.0 ± 5.1	−2.5 ± 1.6
30	No. 25 + (22.6), (22.2)	1.10	22.97//19	−0.14± 0.69	2.9± 2.2	0.6± 2.6	4.8 ± 5.1	−0.7 ± 2.2
31	No. 25 + (21.1), (21.2), (21.5)	1.27	22.87//18	−0.16± 0.64	3.0± 2.8	0.6± 2.6	4.8 ± 5.1	−6.0 ± 2.4
32	No. 25 + (14.2), (23.4), (23.6), (23.7)	0.91	13.93//17	1.76± 0.79	−0.1± 2.9	−0.6± 2.6	5.2 ± 5.2	−1.5 ± 1.7
33	No. 25 + (14.1), (14.3), (14.4), (23.4), (23.6), (23.7)	0.92	12.59//15	0.07± 0.71	2.6± 2.8	−0.5± 2.6	4.9 ± 5.1	−3.3 ± 1.7
34	No. 25 + (14.2), (22.6), (22.2)	1.12	22.40//18	0.35± 0.95	2.1± 3.0	0.3± 2.6	4.9 ± 5.1	−0.7 ± 2.2
35	No. 25 + (14.1), (14.3), (14.4), (22.6), (22.2)	0.89	12.80//16	−1.53± 0.82	5.1± 2.9	1.4± 2.6	4.5 ± 5.1	−0.7 ± 2.2

[a] The numbers given are the differences in ppm between the adjusted values, X_i, and the following exact fiducial values, X_{oi}: $\alpha_o^{-1} = 137.0360$; $e_o = 1.602185 \times 10^{-19}$ C; $K_o = 1.000000$; $N_{Ao} = 6.022020 \times 10^{23}$ mol^{-1}; $(\mu_\mu/\mu_p)_o = 3.183350$. That is, $X_i = X_{oi}\,[1 + \Delta_i \times 10^{-6}]$.

already been explored in table 29.2. Since table 31.1 is more or less self explanatory, we limit ourselves to general remarks intended as a guide to the various adjustments.

Adjustment No. 22, which involves all 31 items of stochastic data listed in table 27.1, and adjustment Nos. 23 and 24, show once again the highly discrepant nature of both the Kaufman, Lamb, et al. $(\Delta E - S)_H$ result for α, and Knowles' measurement of λ_C; the former is deleted in No. 23, and both are excluded in No. 24. Similarly, adjustment Nos. 25 and 26, in which, respectively, both values of F and both values of μ_p'/μ_N have also been deleted, once again show the discrepant nature of the Faraday measurements and the strong interaction between F, μ_p'/μ_N, and the adjusted values of K and N_A (see also sec.III.A.29). Thus, on the basis of this analysis and the analyses discussed in the other sections of part III, we shall exclude from further consideration the four items which have been clearly identified as being discrepant: the Kaufman, Lamb, et al. value of α [eq (23.5)]; the Knowles value of λ_C [eq (18.2)]; and the two NBS Faraday determinations [eqs (13.1) and (13.2)].

The next five adjustments in table 31.1 explore the

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effect of deleting various items of QED data. (Note that in the table, "No. 25 +" means all of the deletions of adjustment No. 25.). In adjustments Nos. 27 and 28, the next two most discrepant QED items (as indicated by their normalized residuals in adjustment No. 25), are separately deleted: the Shyn et al. value of α; and the Chicago-Yale value of ν_{Mhfs}; (r_i = 2.09 for the former, and r_i = 1.30 for the latter). Adjustment No. 29 shows the effect of deleting all four of the hydrogen fine-structure α values, while adjustment No. 30 shows the effect of deleting the two hyperfine splitting determinations [ν_{Mhfs} and α^{-1}(Hhfs)]. Note that in both No. 28 and No. 30, the adjusted value of μ is simply the weighted mean of the three direct determinations. Adjustment No. 31 is of interest because it gives the indirect value of μ as implied by the remaining data when the three direct determinations are deleted. The difference in μ as obtained from this adjustment and either No. 28 or No. 30 is some 6 ppm, relatively large compared with the 2.2 to 2.4 ppm uncertainty in the adjusted values of μ.

The last four adjustments give an indication of the effect of deleting the key data which determine the more precise WQED or indirect values of α, namely, the measurements of γ_p'(low). Of particular interest is a comparison of adjustment No. 32 in which the NBS γ_p'(low) measurement is deleted along with the remaining three highly compatible fine structure α values, and No. 35 in which the highly compatible ETL, NPL, and VNIIM γ_p'(low) measurements are deleted along with the two hyperfine splitting determinations [ν_{Mhfs} and α^{-1}(Hhfs)]. It may be seen that α^{-1} changes by some 3.3 ppm, a relatively large amount compared with its 0.8 ppm uncertainty. This large change arises because the indirect WQED value of α implied by the NBS determination [see eq (31.2a)] is more nearly equal to the hydrogen fine-structure α values than to the hyperfine splitting α values, while for the indirect WQED values of α implied by the ETL, NPL, and VNIIM determinations [see eq (31.2b)], the reverse is true. This situation is also indicated by adjustments Nos. 33 and 34 and which are similar, respectively, to Nos. 35 and 32, but with the fine-structure and hyperfine splitting deletions interchanged. In these, no large shifts in α are observed because the "high value" hyperfine splitting α result is balanced by the "low value" NBS γ_p'(low) α result (No. 33); and the "low value" fine-structure α result is balanced by the "high value" ETL, NPL, and VNIIM γ_p'(low) α result (No. 34).

TABLE 31.2. Summary of least-squares adjustments involving all of the stochastic data with selected expanded uncertainties

Adjust. No.	Eq. Nos. of items deleted	Birge ratio, R_B	χ^2/ν, ν=degrees of freedom	Adjusted values, X_i[a] Δ_i				
				α^{-1}	e	K	N_A	μ_μ/μ_p
γ_p'(low) and x-ray data uncertainties expanded by 1.43 and 1.28, respectively								
36	(23.5), (18.2)	1.13	29.43//23	−0.12 ± 0.69	−2.2 ± 2.3	−4.5 ± 2.0	14.9 ± 3.8	−3.5 ± 1.7
37	(23.5), (18.2), (13.1), (13.2)	0.97	19.65//21	0.03 ± 0.69	3.0 ± 2.8	0.8 ± 2.6	4.1 ± 2.6	−3.3 ± 1.7
38	No. 37 + (14.2), (23.4), (23.6), (23.7)	0.78	10.39//17	1.40 ± 0.88	0.8 ± 2.9	0.0 ± 2.6	4.4 ± 5.1	−1.9 ± 1.8
39	No. 37 + (14.1), (14.3), (14.4), (22.2), (22.6)	0.78	9.71//16	−2.0 ± 1.0	6.2 ± 3.0	2.0 ± 2.6	3.7 ± 5.2	−0.7 ± 2.2
γ_p'(low), x-ray, and QED uncertainties expanded by 1.43, 1.28, and 1.40, respectively								
40	(23.5), (18.2)	1.03	24.42//23	0.04 ± 0.82	−2.4 ± 2.4	−4.6 ± 2.0	15.0 ± 3.8	−3.3 ± 2.3
41	(23.5), (18.2), (13.1), (13.2)	0.83	14.50//21	0.26 ± 0.82	2.6 ± 2.9	0.7 ± 2.6	4.2 ± 5.1	−3.1 ± 2.3
42	No. 41 + (14.2), (23.4), (23.6), (23.7)	0.71	8.55//17	1.8 ± 1.1	0.3 ± 3.1	−0.2 ± 2.6	4.4 ± 5.2	−1.5 ± 2.4
43	No. 41 + (14.1), (14.3), (14.4), (22.2), (22.6)	0.72	8.24//16	−1.5 ± 1.2	5.4 ± 3.2	1.7 ± 2.7	3.8 ± 5.2	−0.7 ± 3.1

[a] See footnote, table 31.1.

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In table 31.2, we summarize some adjustments in which the uncertainties of various quantities have been increased. In the first four adjustments, the *a priori* uncertainties assigned the four low field γ_p' values and those assigned the x-ray data are expanded by the multiplicative factors 1.43 and 1.28, respectively. These are, of course, the same expansion factors used in the previous section when only the WQED data were being studied.[30] These expansions are being considered here for the same reasons as before, namely, to investigate the effect of expanding selected uncertainties, where the motivation for such expansion is to better reflect the variability of the data. Adjustment Nos. 36 through 39 of table 31.2 are the same as Nos. 24, 25, 32, and 35 of table 31.1 with the exception of the increased uncertainties. In general, χ^2 or R_B is significantly lower for these new adjustments, some of the uncertainties of the adjusted values are increased, and their numerical values are slightly changed.

Adjustment Nos. 40 through 43 show the effect of expanding the *a priori* uncertainties assigned all of the QED data as well as the γ_p'(low) and x-ray measurements. The multiplicative factor used for the QED data, 1.40, is the Birge ratio of the weighted mean of all of the QED α values except that implied by the Kaufman, Lamb, et al. measurement of $(\Delta E - S)_H$ (see line 7 of table 30.1). We use this larger, more conservative factor rather than, for example, the 1.19 implied by adjustment No. 2 of table 30.2, because of the large variability of the QED data and the fact that the excellent agreement among the three direct determinations of μ_μ/μ_p biases R_B in adjustment No. 2 of table 30.2 to the low side.

Adjustment Nos. 40 through 43 are identical to adjustment Nos. 36 through 39 of the same table with the exception of the expanded QED uncertainties; and with Nos. 24, 25, 32, and 35 of table 31.1 with the exception of the expanded γ_p'(low), x-ray, and QED uncertainties. Again we see the effect of the uncertainty expansion is to decrease R_B or χ^2, to increase some of the uncertainties of the adjusted values, and to change the numerical values of the latter by only small fractions of their uncertainties. This becomes particularly clear upon comparing adjustment No. 41 of the table with No. 25 of table 31.1.

In conclusion, we believe that adjustment No. 41 represents the most reasonable way of handling all of the stochastic data. The increased uncertainties in the adjusted constants resulting from expanding the *a priori* uncertainties of the γ_p'(low), x-ray, and QED data more nearly reflect the overall variability of the values of the adjusted constants with the particular selection of input data. The normalized residuals of the 27 items of input data are less than unity, with four exceptions: The ETL γ_p'(low) result, the NPL γ_p' (high) result, the 1931 Bearden-Henins value of Λ, and the Shyn et al. value of α [eqs (14.2), (14.6), (16.3), and (23.6)]. For these, r_i is 1.07, 1.23, 1.17, and 1.49, respectively. In the worst case, that for Shyn et al., the input value differs from the adjusted value by only one and a half standard deviations. The overall value of $\chi^2 = 14.50$ for the adjustment (21 degrees of freedom) is quite satisfactory (R_B=0.83). We do not believe that our approach is too conservative as one might assume at first glance from this value of χ^2 because three groups of data are in such abnormally good agreement among themselves that their own internal χ^2 contributes very little to the overall χ^2 (K, μ_p'/μ_N, and μ_μ/μ_p; see tables 28.1 and 30.1). Indeed, if adjustments are carried out in which we use eleven input equations, each representing the weighted mean of each of the different kinds of data in table 27.1 (excluding Knowles λ_C result, the two Faraday measurements, and the Kaufman, Lamb, et al. α value), then we find χ^2=12.01 for the adjustment corresponding to No. 25 (R_B=1.55); χ^2=7.64 for the adjustment corresponding to No. 37 (R_B=1.24); and χ^2=6.67 for the adjustment corresponding to No. 41 (R_B=1.15). In each, the number of degrees of freedom is 11 − 6 = 5. Since χ^2 for such adjustments depends *only* on the compatibility of *dissimilar* kinds of data rather than on the compatability of *both similar and dissimilar* data, the 6.67 value for χ^2 for 5 degrees of freedom gives perhaps a clearer picture of the general agreement of the data used in our recommended adjustment, No. 41.[31] (The probability for ν=5 that a value of χ^2 as large or larger than 6.67 can occur by chance is 0.25.)

The entire set of constants resulting from adjustment No. 41, which is our recommended set, will be given in section IV.B.33.

IV. Recommended Values of Fundamental Constants

In this portion of the paper we give numerical values for the physical constants comprising both our "best" set of WQED constants, and our final recommended set derived from the WQED and QED data together. A detailed description of how these constants were obtained from the five or six adjustable constants or "unknowns" used in each of these two adjustments will be given as well. For our recommended set of fundamental constants we also present an expanded variance-covariance and correlation coefficient matrix from which the uncertainty of any combination of constants not given in the tables may be readily calculated, and a brief description of how the matrix is to be used. For the WQED set, we simply give an unexpanded version of this matrix.

[30] Adjustment No. 25 in table 31.3 yields the same x-ray expansion factor as does the adjustment identical to No. 25 but with the *a priori* uncertainties assigned the γ_p'(low) measurements expanded by the multiplicative factor 1.43. The fact that the x-ray data are only weakly coupled to the other data accounts for the constancy of the factor 1.28.

[31] See footnote 29, section III.A.29.

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TABLE 32.1. Our best set of WQED constants based on adjustment No. 21 of table 29.3. $\chi^2 = 8.75$ for $18 - 5 = 13$ degrees of freedom; $R_B = 0.82$.[a]

Quantity	Symbol	Value	Uncertainty (ppm)	Units SI	Units cgs[b]
Speed of light in vacuum	c	299792458(1.2)	0.004	$m \cdot s^{-1}$	10^2 $cm \cdot s^{-1}$
Fine-structure constant, $[\mu_0 c^2/4\pi](e^2/\hbar c)$	α α^{-1}	7.2973461(81) 137.03612(15)	1.1 1.1	10^{-3}	10^{-3}
Elementary charge	e	1.6021876(50) 4.803238(15)	3.1 3.1	10^{-19} C	10^{-20} emu 10^{-10} esu
Planck constant	h $\hbar = h/2\pi$	6.626167(38) 1.0545872(60)	5.7 5.7	10^{-34} J·s 10^{-34} J·s	10^{-27} erg·s 10^{-27} erg·s
Avogadro constant	N_A	6.022046(31)	5.2	10^{23} mol^{-1}	10^{23} mol^{-1}
Electron rest mass	m_e	9.109533(47) 5.4858026(21)	5.1 0.38	10^{-31} kg 10^{-4} u	10^{-28} g 10^{-4} u
Proton rest mass	m_p	1.6726483(86) 1.007276470(11)	5.2 0.011	10^{-27} kg u	10^{-24} g u
Ratio of proton mass to electron mass	m_p/m_e	1836.15152(70)	0.38		
Neutron rest mass	m_n	1.6749541(86) 1.008665012(37)	5.2 0.037	10^{-27} kg u	10^{-24} g u
Josephson frequency-voltage ratio	$2e/h$	4.835941(13)	2.7	10^{14} $Hz \cdot V^{-1}$	
Quantum of circulation	$h/2m_e$ h/m_e	3.6369410(80) 7.273882(16)	2.2 2.2	10^{-4} $J \cdot s \cdot kg^{-1}$ 10^{-4} $J \cdot s \cdot kg^{-1}$	$erg \cdot s \cdot g^{-1}$ $erg \cdot s \cdot g^{-1}$
Faraday constant, $N_A e$	F	9.648447(29) 2.8925317(87)	3.0 3.0	10^4 $C \cdot mol^{-1}$	10^3 $emu \cdot mol^{-1}$ 10^{14} $esu \cdot mol^{-1}$
Bohr radius, $[\mu_0 c^2/4\pi]^{-1}(\hbar^2/m_e e^2) = \alpha/4\pi R_\infty$	a_0	5.2917673(58)	1.1	10^{-11} m	10^{-9} cm
Classical electron radius, $[\mu_0 c^2/4\pi](e^2/m_e c^2) = \alpha^3/4\pi R_\infty$	r_e	2.8179328(93)	3.3	10^{-15} m	10^{-13} cm
Gyromagnetic ratio of protons in H_2O	γ'_p $\gamma'_p/2\pi$	2.6751278(80) 4.257598(13)	3.0 3.0	10^8 $s^{-1} \cdot T^{-1}$ 10^7 $Hz \cdot T^{-1}$	10^4 $s^{-1} \cdot G^{-1}$ 10^3 $Hz \cdot G^{-1}$
Magnetic moment of protons in H_2O in nuclear magnetons	μ'_p/μ_N	2.7927740(11)	0.38		
Ratio of 1 January 1969 BIPM ampere to SI ampere	$K \equiv A_{BI69}/A$	1.0000003(26)	2.6		
Ratio of 1 January 1969 BIPM ohm to SI ohm	$\bar{R} \equiv \Omega_{BI69}/\Omega$	0.99999946(19)	0.19		
Ratio, kx-unit to ångström, $\Lambda = \lambda(\text{Å})/\lambda(\text{kxu})$; $\lambda(\text{Cu}K\alpha_1) \equiv 1.537400$ kxu	Λ	1.0020771(54)	5.3		

[a, b] See footnotes, table 33.1.

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A. The WQED Values

Our best set of WQED constants follows from adjustment No. 21, table 29.3. To reiterate, this adjustment includes all of the WQED data listed in table 27.1, that is, the first 21 items in the table [eqs (4.4) through (18.3)] but with the following deletions and modifications:

(1) The Knowles measurement of the electron compton wavelength, λ_C [eq (18.2)], is deleted because of its high degree of inconsistency with the remaining data (see sec. III.A.29).

(2) The two NBS measurements of the Faraday, F, by Craig et al., and by Marinenko and Taylor [eqs (13.1) and (13.2)] are deleted because of their incompatibility with the remaining data (see sec. III.A.29).

(3) The uncertainties assigned the four low field determinations of the proton gyromagnetic ratio, γ'_p, at ETL, NBS, NPL, and VNIIM [eqs (14.1), (14.2), (14.3), (14.4)] are increased by the multiplicative factor 1.43 in order to make the γ'_p(low) values more compatible and to better reflect their overall agreement (see secs. III.A.28 and 29).

(4) The uncertainties assigned the six items of x-ray data used in adjustment No. 21 are increased by the multiplicative factor 1.28 for the same reasons as in (3). These data are the Bearden and I. Henins, A. Henins, and Spijkerman and Bearden values of the ratio of the kxu to the metre, Λ [eqs (16.3), (16.5), (16.7)]; the I. Henins and Bearden, and Bearden values of $N_A\Lambda^3$ [eqs (17.1), (17.2)]; and the Van Assche et al. value of λ_C [eq (18.3)].

For this adjustment, $\chi^2 = 8.75$ for $18-5=13$ degrees of freedom and the Birge ratio, R_B, equals 0.82. The five unknowns or adjustable constants were taken to be α^{-1}, $K \equiv A_{BI69}/A$, N_A, $\bar{R} \equiv \Omega_{BI69}/\Omega$, and Λ. The numerical values for these constants and those we have derived from them, including the relevant uncertainties and variance matrix, are given in the following section.

TABLE 32.2. Combined variance—covariance and correlation coefficient matrix for our best WQED constants. The variances and covariances, which are, respectively, on and above the main diagonal, are expressed in (parts per million)^2. The correlation coefficients are in *italics* below the diagonal.

	α^{-1}	K^a	N_A	$\bar{R}^b$	Λ
α^{-1}	1.218	−0.719	0.256	−0.018	−0.104
K^a	*−0.246*	6.997	−13.273	−0.000	3.498
N_A	*0.045*	*−0.974*	26.540	−0.054	−6.958
$\bar{R}^b$	*−0.086*	*−0.001*	*−0.055*	0.036	0.015
Λ	*−0.018*	*0.247*	*−0.253*	*0.014*	28.548

[a] $K \equiv A_{BI69}/A$. [b] $\bar{R} \equiv \Omega_{BI69}/\Omega$.

32. WQED Set and Variance Matrix

Table 32.1 gives the WQED constants based on adjustment No. 21 of table 29.3. Table 32.2 gives the combined variance-covariance and correlation coefficient matrix for this set of constants. Since the set is intended for use only in the particular instance when one needs values which are independent of quantum electrodynamics, we do not give an exhaustive list. Rather the list is restricted to those quantities which are most likely to be used under these conditions. Quantities which are not listed here may of course be computed from those which are given; additional auxiliary constants may be obtained from table 11.1.

One reason for not including a complete list of WQED constants is the smallness of the differences between the output values of the WQED adjustment and the full adjustment. Only for the fine-structure constant itself is the difference larger than half a standard deviation; in most cases it is much smaller. This is apparent from table 32.3 in which we compare selected WQED values with the corresponding final recommended values. Clearly for the general user there is no significant advantage to using the WQED

TABLE 32.3. A Comparison of our best WQED values of selected constants (table 32.1) with our final recommended values (table 33.1)

Quantity	Units	WQED value and ppm uncertainty		Final recommended value and ppm uncertainty		Difference (WQED - recommended) (ppm)
α^{-1}		137.03612(15)	1.1	137.03604(11)	0.82	+0.6
e	10^{-19} C	1.6021876(50)	3.1	1.6021892(46)	2.9	−1.0
h	10^{-34} J·s	6.626167(38)	5.7	6.626176(36)	5.4	−1.4
m_e	10^{-31} kg	9.109533(47)	5.1	9.109534(47)	5.1	−0.1
N_A	10^{23} mol^{-1}	6.022046(31)	5.2	6.022045(31)	5.0	−0.2
μ'_p/μ_N		2.7927740(11)	0.38	2.7927740(11)	0.38	0.0
F	10^4 C·mol^{-1}	9.648447(29)	3.0	9.648456(27)	2.8	−0.9
Λ		1.0020771(54)	5.3	1.0020772(54)	5.3	−0.1

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set of constants in preference to the recommended set, even if one has misgivings concerning the validity or accuracy of quantum electrodynamics. For the special use of those who wish to verify QED theory and hence require constants which are free of the theory which is being tested, the fine-structure constant, α, is often the only one needed.[32]

We shall postpone our detailed discussion of how the various quantities in table 32.1 were obtained from the adjusted values of α^{-1}, K, N_A, $\bar{R}$, and Λ, and how the matrix given in table 32.2 is to be used, until the next section where we present our final recommended set of constants.

B. The Recommended Values

Our final recommended set of constants follows from adjustment No. 41, table 31.2. To reiterate, this adjustment includes *all* of the data listed in table 27.1, except for the following deletions and modifications in addition to those made to the data used for the WQED adjustment of the previous section:

(1) The Kaufman, Lamb, et al. $(\Delta E-\mathcal{S})_H$ result for α [eq (23.5)] is deleted because of its gross disagreement with the other data.

(2) The uncertainties assigned the remaining QED data are expanded by the multiplicative factor 1.40 for the same reasons as for the γ'_p(low) and x-ray data (see sec. IV.A.). These QED data are the value of α derived from the Wesley-Rich (as revised by Granger and Ford) electron anomalous moment determination [eq (19.8)]; the hydrogen hfs value of α derived from the measurement of ν_{Hhfs} using the hydrogen maser [eq (22.6)]; the determinations of the ratio of the magnetic moment of the muon to that of the proton, μ_μ/μ_p, by Crowe, Williams, et al. [eq (21.1)], Hutchinson, et al. [eq (21.2)], and DeVoe, Telegdi, et al. (as revised by Jarecki and Herman) [eq (21.5)]; the Chicago-Yale value of ν_{Mhfs} [eq (21.2)]; the Baird et al. ΔE_H result for α [eq (23.4)]; and the two $(\Delta E-\mathcal{S})_H$ α values of Shyn et al. [eq (23.6)] and of Cosens and Vorburger [eq (23.7)].

For this adjustment, $\chi^2 = 14.50$ for $27-6 = 21$ degrees of freedom, and the Birge ratio, R_B, equals 0.83. The six unknowns or adjustable constants were taken to be α^{-1}, $K\equiv A_{Bl69}/A$, N_A, $\bar{R}\equiv\Omega_{Bl69}/\Omega$, Λ, and $\mu\equiv\mu_\mu/\mu_p$. The numerical values for these constants and those we have derived from them, including the relevant uncertainties and variance matrix, are given in the following section.

33. Final Recommended Set and Variance Matrix

Tables 33.1, 33.2, and 33.3 give our recommended set of constants based on adjustment No. 41 of Table 31.2. However, the following quantities, which are also listed in the tables, were not subject to adjustment in any way and were taken directly from table 11.1; c; M_p; R_∞; $g_e/2 = \mu_e/\mu_B$; $g_\mu/2$; μ'_p/μ_B; μ_p/μ_B; μ_e/μ_p; and $(2e/h)_{Bl69}$. Similarly, the atomic mass of the neutron, M_n, was taken from table 9.1; the Newtonian gravitational constant, G, from eq (24.5); the molar volume of an ideal gas, V_m, from eq (25.2); and the gas constant, R, from eq (25.4).

Of course, the six quantities α^{-1}, $K\equiv A_{Bl69}/A$, $\bar{R}\equiv\Omega_{Bl69}/\Omega$, N_A, Λ, and $\mu\equiv\mu_\mu/\mu_p$ follow directly from the adjustment itself since these constants were the unknowns or adjustable constants used therein. The other constants follow from appropriate combinations of these basic six and the auxiliary constants given in table 11.1. The elementary charge is calculated from the equation

$$e = [(\mu_0 c/4)(2e/h)_{Bl69}]^{-1}\cdot\alpha K\bar{R}. \qquad (33.1)$$

The remaining constants of interest may then be expressed as[33]

$$h = (\mu_0\, c/2)\cdot\alpha^{-1}\, e^2;$$

$$m_e = \mu_0 R_\infty\cdot\alpha^{-3}\, e^2;$$

[32] It should be noted, however, that the weighted average of the three direct measurements of μ_μ/μ_p given in table 21.1, $\mu_\mu/\mu_p = 3.1833479(80)$ (2.2 ppm), exceeds our final recommended value, $\mu_\mu/\mu_p = 3.1833402(72)$ (2.3 ppm), by 2.4 ppm.

[33] We have not included the constants for which the appropriate equations are already given in the Tables, for example, $F = N_A e$, $a_0 = \alpha/4\pi R_\infty$, or which are simple ratios such as m_p/m_e, e/m_e, etc. Note that all quantities to the left of · are auxiliary constants assumed to be exactly known.

TABLE 33.1. Our final recommended set of constants based on adjustment No. 41 of table 31.2. $\chi^2 = 14.50$ for $27 - 6 = 21$ degrees of freedom; $R_B = 0.83$[a]

Quantity	Symbol	Value	Uncertainty (ppm)	Units SI[b]	Units cgs[c]
Speed of light in vacuum	c	299792458(1.2)	0.004	$m\cdot s^{-1}$	10^2 $cm\cdot s^{-1}$
Permeability of vacuum	μ_0	4π -12.5663706144		10^{-7} $H\cdot m^{-1}$ 10^{-7} $H\cdot m^{-1}$	
Permittivity of vacuum, $1/\mu_0 c^2$	ϵ_0	8.854187818(71)	0.008	10^{-12} $F\cdot m^{-1}$	
Fine-structure constant, $[\mu_0 c^2/4\pi](e^2/\hbar c)$	α α^{-1}	7.2973506(60) 137.03604(11)	0.82 0.82	10^{-3}	10^{-3}

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TABLE 33.1. Our final recommended set of constants based on adjustment No. 41 of table 31.2. $\chi^2 = 14.50$ for $27 - 6 = 21$ degrees of freedom; $R_B = 0.83$[a]—Continued

Quantity	Symbol	Value	Uncertainty (ppm)	Units SI[b]	Units cgs[c]
Elementary charge	e	1.6021892(46)	2.9	10^{-19} C	10^{-20} emu
		4.803242(14)	2.9		10^{-10} esu
Planck constant	h	6.626176(36)	5.4	10^{-34} J·s	10^{-27} erg·s
	$\hbar = h/2\pi$	1.0545887(57)	5.4	10^{-34} J·s	10^{-27} erg·s
Avogadro constant	N_A	6.022045(31)	5.1	10^{23} mol^{-1}	10^{23} mol^{-1}
Atomic mass unit, 10^{-3} kg·mol^{-1} N_A^{-1}	u	1.6605655(86)	5.1	10^{-27} kg	10^{-24} g
Electron rest mass	m_e	9.109534(47)	5.1	10^{-31} kg	10^{-28} g
		5.4858026(21)	0.38	10^{-4} u	10^{-4} u
Proton rest mass	m_p	1.6726485(86)	5.1	10^{-27} kg	10^{-24} g
		1.007276470(11)	0.011	u	u
Ratio of proton mass to electron mass	m_p/m_e	1836.15152(70)	0.38		
Neutron rest mass	m_n	1.6749543(86)	5.1	10^{-27} kg	10^{-24} g
		1.008665012(37)	0.037	u	u
Electron charge to mass ratio	e/m_e	1.7588047(49)	2.8	10^{11} C·kg^{-1}	10^{7} emu·g^{-1}
		5.272764(15)	2.8		10^{17} esu·g^{-1}
Magnetic flux quantum, $[c]^{-1}$ $(hc/2e)$	Φ_0	2.0678506(54)	2.6	10^{-15} Wb	10^{-7} G·cm^2
	h/e	4.135701(11)	2.6	10^{-15} J·s·C^{-1}	10^{-7} erg·s·emu^{-1}
		1.3795215(36)	2.6		10^{-17} erg·s·esu^{-1}
Josephson frequency-voltage ratio	$2e/h$	4.835939(13)	2.6	10^{14} Hz·V^{-1}	
Quantum of circulation	$h/2m_e$	3.6369455(60)	1.6	10^{-4} J·s·kg^{-1}	erg·s·g^{-1}
	h/m_e	7.273891(12)	1.6	10^{-4} J·s·kg^{-1}	erg·s·g^{-1}
Faraday constant, $N_A e$	F	9.648456(27)	2.8	10^{4} C·mol^{-1}	10^{3} emu·mol^{-1}
		2.8925342(82)	2.8		10^{14} esu·mol^{-1}
Rydberg constant, $[\mu_0 c^2/4\pi]^2$ $(m_e e^4/4\pi\hbar^3 c)$	R_∞	1.097373177(83)	0.075	10^{7} m^{-1}	10^{5} cm^{-1}
Bohr radius, $[\mu_0 c^2/4\pi]^{-1}$ $(\hbar^2/m_e e^2) = \alpha/4\pi R_\infty$	a_0	5.2917706(44)	0.82	10^{-11} m	10^{-9} cm
Classical electron radius, $[\mu_0 c^2/4\pi]$ $(e^2/m_e c^2) = \alpha^3/4\pi R_\infty$	$r_e = \alpha\lambda\!\!\!^{-}_C$	2.8179380(70)	2.5	10^{-15} m	10^{-13} cm
Thomson cross section, $(8/3)\pi r_e^2$	σ_e	0.6652448(33)	4.9	10^{-28} m^2	10^{-24} cm^2
Free electron g-factor, or electron magnetic moment in Bohr magnetons	$g_e/2 = \mu_e/\mu_B$	1.0011596567(35)	0.0035		
Free muon g-factor, or muon magnetic moment in units of $[c]$ $(e\hbar/2m_\mu c)$	$g_\mu/2$	1.00116616(31)	0.31		
Bohr magneton, $[c]$ $(e\hbar/2m_e c)$	μ_B	9.274078(36)	3.9	10^{-24} J·T^{-1}	10^{-21} erg·G^{-1}
Electron magnetic moment	μ_e	9.284832(36)	3.9	10^{-24} J·T^{-1}	10^{-21} erg·G^{-1}
Gyromagnetic ratio of protons in H_2O	γ'_p	2.6751301(75)	2.8	10^{8} s^{-1}·T^{-1}	10^{4} s^{-1}·G^{-1}
	$\gamma'_p/2\pi$	4.257602(12)	2.8	10^{7} Hz·T^{-1}	10^{3} Hz·G^{-1}
γ'_p corrected for diamagnetism of H_2O	γ_p	2.6751987(75)	2.8	10^{8} s^{-1}·T^{-1}	10^{4} s^{-1}·G^{-1}
	$\gamma_p/2\pi$	4.257711(12)	2.8	10^{7} Hz·T^{-1}	10^{3} Hz·G^{-1}

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TABLE 33.1. Our final recommended set of constants based on adjustment No. 41 of table 31.2. $\chi^2 = 14.50$ for 27 − 6 = 21 degrees of freedom; $R_B = 0.83$[a]—Continued

Quantity	Symbol	Value	Uncertainty (ppm)	Units SI[b]	Units cgs[c]
Magnetic moment of protons in H_2O in Bohr magnetons	μ'_p/μ_B	1.52099322(10)	0.066	10^{-3}	10^{-3}
Proton magnetic moment in Bohr magnetons	μ_p/μ_B	1.521032209(16)	0.011	10^{-3}	10^{-3}
Ratio of electron and proton magnetic moments	μ_e/μ_p	658.2106880(66)	0.010		
Proton magnetic moment	μ_p	1.4106171(55)	3.9	10^{-26} J·T^{-1}	10^{-23} erg·G^{-1}
Magnetic moment of protons in H_2O in nuclear magnetons	μ'_p/μ_N	2.7927740(11)	0.38		
μ'_p/μ_N corrected for diamagnetism of H_2O	μ_p/μ_N	2.7928456(11)	0.38		
Nuclear magneton, $[c](e\hbar/2m_pc)$	μ_N	5.050824(20)	3.9	10^{-27} J·T^{-1}	10^{-24} erg·G^{-1}
Ratio of muon and proton magnetic moments	μ_μ/μ_p	3.1833402(72)	2.3		
Muon magnetic moment	μ_μ	4.490474(18)	3.9	10^{-26} J·T^{-1}	10^{-23} erg·G^{-1}
Ratio of muon mass to electron mass	m_μ/m_e	206.76865(47)	2.3		
Muon rest mass	m_μ	1.883566(11) 0.11342920(26)	5.6 2.3	10^{-28} kg u	10^{-25} g u
Compton wavelength of the electron, $h/m_ec = \alpha^2/2R_\infty$	λ_C $\lambda\!\!\!^{-}_C = \lambda_C/2\pi = \alpha a_0$	2.4263089(40) 3.8615905(64)	1.6 1.6	10^{-12} m 10^{-13} m	10^{-10} cm 10^{-11} cm
Compton wavelength of the proton, h/m_pc	$\lambda_{C,p}$ $\lambda\!\!\!^{-}_{C,p} = \lambda_{C,p}/2\pi$	1.3214099(22) 2.1030892(36)	1.7 1.7	10^{-15} m 10^{-16} m	10^{-13} cm 10^{-14} cm
Compton wavelength of the neutron, h/m_nc	$\lambda_{C,n}$ $\lambda\!\!\!^{-}_{C,n} = \lambda_{C,n}/2\pi$	1.3195909(22) 2.1001941(35)	1.7 1.7	10^{-15} m 10^{-16} m	10^{-13} cm 10^{-14} cm
Molar volume of ideal gas at s.t.p.	V_m	22.41383(70)	31	10^{-3} m^3·mol^{-1}	10^3 cm^3·mol^{-1}
Molar gas constant, p_0V_m/T_0 ($T_0 \equiv 273.15$ K; $p_0 \equiv 101325$Pa $\equiv 1$ atm)	R	8.31441(26) 8.20568(26)	31 31	J·mol^{-1}·K^{-1} 10^{-5} m^3·atm·mol^{-1}·K^{-1}	10^7 erg·mol^{-1}·K^{-1} 10 cm^3·atm·mol^{-1}·K^{-1}
Boltzmann constant, R/N_A	k	1.380662(44)	32	10^{-23} J·K^{-1}	10^{-16} erg·K^{-1}
Stefan-Boltzmann constant, $\pi^2k^4/60\hbar^3c^2$	σ	5.67032(71)	125	10^{-8} W·m^{-2}·K^{-4}	10^{-5} erg·s^{-1}·cm^{-2}·K^{-4}
First radiation constant, $2\pi hc^2$	c_1	3.741832(20)	5.4	10^{-16} W·m^2	10^{-5} erg·cm^2·s^{-1}
Second radiation constant, hc/k	c_2	1.438786(45)	31	10^{-2} m·K	cm·K
Gravitational constant	G	6.6720(41)	615	10^{-11} m^3·s^{-2}·kg^{-1}	10^{-8} cm^3·s^{-2}·g^{-1}

[a] Note that the numbers in parentheses are the one standard-deviation uncertainties in the last digits of the quoted value computed on the basis of internal consistency, that the unified atomic mass scale $^{12}C \triangleq 12$ has been used throughout, that u=atomic mass unit, C=coulomb, F=farad, G = gauss, H = henry, Hz = hertz = cycle/s, J = joule, K = kelvin (degree Kelvin), Pa = pascal = N·m^{-2}, T = tesla (10^4 G), V = volt, Wb = weber = T·m^2, and W = watt. In cases where formulas for constants are given (e.g., R_∞), the relations are written as the product of two factors. The second factor, in parentheses, is the expression to be used when all quantities are expressed in cgs units, with the electron charge in electrostatic units. The first factor, in brackets, is to be included only if all quantities are expressed in SI units. We remind the reader that with the exception of the auxiliary constants which have been taken to be exact, the uncertainties of these constants are correlated, and therefore the general law of error propagation must be used in calculating additional quantities requiring two or more of these constants. (See text.)

[b] Quantities given in u and atm are for the convenience of the reader; these units are not part of the Système International d'Unité (SI).

[c] In order to avoid separate columns for "electromagnetic" and "electrostatic" units, both are given under the single heading "cgs Units." When using these units, the elementary charge e in the second column should be understood to be replaced by e_m or e_e, respectively.

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TABLE 33.2. Our final recommended values for various quantities involving BIPM as-maintained electrical units (specifically, 1 January 1969 units), the kilo-x-unit (kxu), and the angstrom-star (Å*)[a]

Quantity	Symbol	Value	Uncertainty (ppm)	Units
Ratio of 1 January 1969 BIPM ampere to SI ampere	$K \equiv A_{BI69}/A$	1.0000007(26)	2.6	
Ratio of 1 January 1969 BIPM ohm to SI ohm	$\bar{R} \equiv \Omega_{BI69}/\Omega$	0.99999947(19)	0.19	
Ratio of 1 January 1969 BIPM volt to SI volt	V_{BI69}/V	1.0000002(26)	2.6	
Josephson frequency-voltage ratio used to define V_{BI69}	$(2e/h)_{BI69}$	4.83594000	by definition	10^{14} Hz·V_{BI69}^{-1}
Ratio, kx-unit to ångström, $\Lambda = \lambda(Å)/\lambda(kxu)$; $\lambda(CuK\alpha_1) \equiv 1.537400$ kxu	Λ	1.0020772(54)	5.3	
Ratio, Å* to ångström, $\Lambda^* = \lambda(Å)/\lambda(Å^*)$; $\lambda(WK\alpha_1) \equiv 0.2090100$ Å*	Λ^*	1.0000205(56)	5.6	
Voltage-wavelength conversion product, $V\lambda = hc/e$	$V\lambda$(kxu) $V\lambda$(Å*)	1.2372820(66) 1.2398266(70)	5.3 5.6	10^4 V·kxu 10^4 V·Å*
Compton wavelength of the electron, h/m_ec	λ_C(kxu) λ_C(Å*)	24.21279(13) 24.26259(14)	5.6 5.9	10^{-3} kxu 10^{-3} Å*

[a] See footnote a, table 33.1.

$$M_e = (10^{-3}\ \mathrm{kg}\cdot\mathrm{mol}^{-1})^{-1}\cdot m_eN_A;$$

$$m_p = (10^{-3}\ \mathrm{kg}\cdot\mathrm{mol}^{-1})\, M_p\cdot N_A^{-1};$$

$$m_n = (10^{-3}\ \mathrm{kg}\cdot\mathrm{mol}^{-1})\, M_n\cdot N_A^{-1};$$

$$\gamma_p' = [(\mu_p'/\mu_B)/\mu_0R_\infty]\cdot\alpha^3e^{-1};$$

$$\mu_p'/\mu_N = [10^{-3}\ \mathrm{kg}\cdot\mathrm{mol}^{-1}\, M_p(\mu_p'/\mu_B)/\mu_0R_\infty]\cdot\alpha^3e^{-2}N_A^{-1};$$

$$m_\mu/m_e = (\mu_e/\mu_p)(g_\mu/g_e)\cdot\mu^{-1};$$

$$m_\mu = (m_\mu/m_e)m_e;\ \text{and}$$

$$M_\mu = (10^{-3}\ \mathrm{kg}\cdot\mathrm{mol}^{-1})^{-1}\cdot m_\mu N_A.$$

For the electrical and x-ray quantities we have

$$V_{BI69}/V = K\bar{R};$$

$$\Lambda^* = \Lambda/1.0020567(18);$$

$$V\lambda(\mathrm{kxu}) = hc/e\Lambda;\ V\lambda(Å^*) = hc/e\Lambda^*;$$

and

$$\lambda_C(\mathrm{kxu}) = h/m_ec\Lambda;\ \lambda_C(Å^*) = h/m_ec\Lambda^*.$$

The quantity Λ^* is the ratio of the ångström star to ångström (10^{-10} metre):

$$\Lambda^* = \lambda(Å)/\lambda(Å^*), \tag{33.2}$$

where Å* is the basic unit of the x-ray scale developed by Bearden [33.1, 16.2] and is defined by $\lambda(WK\alpha_1) \equiv 0.2090100$Å*. The relationship given above between Λ and Λ^* then follows directly from this definition and the wavelength of $WK\alpha_1$ based on $\lambda(CuK\alpha_1) \equiv 1.537400$ kxu as given in eq (16.29).

For the energy conversion factors, we have

$$1\ \mathrm{kg} = \{c^2\}\mathrm{J} = \{c^2/e\}\mathrm{eV};\ 1\mathrm{u} = \{10^{-3}c^2/F\}\mathrm{eV};$$

$$1\ \mathrm{eV} = \{e/h\}\mathrm{Hz} = \{e/hc\}\mathrm{m}^{-1} = \{e/k\}\mathrm{K} = \{e\}\mathrm{J},$$

where, as in table 29.1, the braces indicate the numerical value only.

We now turn our attention to a very brief discussion of how the uncertainties assigned the various quantities in the tables were obtained.[34] It must first be remembered that the uncertainties of the adjustable constants which result from a least-squares adjustment are in general correlated. This requires knowledge not only of the variances (squares of the standard deviations) of the adjusted constants, but their covariances as well. The variances are given by the diagonal elements of the error matrix $\mathbf{G}^{-1}$ (a necessary by-product of an adjustment), and the covariances by the off diagonal elements. The variance of the ith adjusted

[34] This discussion follows very closely that given in Appendix A of ref. [0.1]; see also ref. [29.2], p. 233.

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TABLE 33.3. Our final recommended values for various energy conversion factors and equivalents[a]

Quantity		Value	Units	Uncertainty (ppm)
1 kilogram (kgc^2)		5.609545(16)	10^{29} MeV	2.9
1 Atomic mass unit (uc^2)		931.5016(26)	MeV	2.8
1 Electron mass (m_ec^2)		0.5110034(14)	MeV	2.8
1 Muon mass ($m_\mu c^2$)		105.65948(35)	MeV	3.3
1 Proton mass (m_pc^2)		938.2796(27)	MeV	2.8
1 Neutron mass (m_nc^2)		939.5731(27)	MeV	2.8
1 Electron volt		1.6021892(46)	10^{-19} J	2.9
			10^{-12} erg	2.9
	leV/h	2.4179696(63)	10^{14} Hz	2.6
	leV/hc	8.065479(21)	10^5 m^{-1}	2.6
			10^3 cm^{-1}	2.6
	leV/k	1.160450(36)	10^4 K	31
Voltage-wavelength conversion, hc		1.2398520(32)	10^{-6} eV·m	2.6
			10^{-4} eV·cm	2.6
Rydberg constant	$R_\infty hc$	2.179907(12)	10^{-18} J	5.4
			10^{-11} erg	5.4
		13.605804(36)	eV	2.6
	$R_\infty c$	3.28984200(25)	10^{15} Hz	0.075
	$R_\infty hc/k$	1.578885(49)	10^5 K	31
Bohr magneton	μ_B	5.7883785(95)	10^{-5} eV·T^{-1}	1.6
	μ_B/h	1.3996123(39)	10^{10} Hz·T^{-1}	2.8
	μ_B/hc	46.68604(13)	m^{-1}·T^{-1}	2.8
			10^{-2} cm^{-1}·T^{-1}	2.8
	μ_B/k	0.671712(21)	K·T^{-1}	31
Nuclear magneton	μ_N	3.1524515(53)	10^{-8} eV·T^{-1}	1.7
	μ_N/h	7.622532(22)	10^6 Hz·T^{-1}	2.8
	μ_N/hc	2.5426030(72)	10^{-2} m^{-1}·T^{-1}	2.8
			10^{-4} cm^{-1}·T^{-1}	2.8
	μ_N/k	3.65826(12)	10^{-4} K·T^{-1}	31

[a] See footnote a, table 33.1, and text.

constant, $v_{ii} = \epsilon_i^2$, is thus equal to $(\mathbf{G}^{-1})_{ii}$, while the covariance of the ith and jth adjusted constants, v_{ij}, is given by $(\mathbf{G}^{-1})_{ij}$. The error matrix is symmetrical so that $v_{ij} = v_{ji}$.

If a quantity Q_k depends on N statistically correlated quantities x_i according to the equation

$$Q_k = Q_k(x_1, x_2 \ldots x_N), \tag{33.3}$$

then the variance in Q_k, ϵ_k^2, is given by

$$\epsilon_k^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial Q_k}{\partial x_i} \frac{\partial Q_k}{\partial x_j} v_{ij}, \tag{33.4}$$

where v_{ij} is the covariance of x_i and x_j. This is a completely general form. The units of ϵ_k are the same as the units of Q_k and the units of v_{ij} are the product of the units of x_i and x_j. Often it is more convenient to express the variances and covariances in relative (dimensionless) units, for example, in percent or ppm.

For most cases of interest involving the fundamental constants, Q_k will depend on a number of constants Z_j as a product of powers:

$$Q_k = q_k \prod_{j=1}^{J} Z_j^{Y_{kj}} \tag{33.5}$$

(q_k is just a numerical factor). If the variances and covariances are then expressed in relative units, eq (33.4) becomes

$$\epsilon_k^2 = \sum_{i=1}^{J} \sum_{j=1}^{J} Y_{ki} Y_{kj} v_{ij}, \tag{33.6}$$

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TABLE 33.4. Expanded and combined variance-covariance and correlation coefficient matrix for our final recommended set of constants. The variances and covariances, which are, respectively, on and above the main diagonal, are in (parts per million)^2. The correlation coefficients are in *italics* below the diagonal

	α^{-1}	K^a	N_A	$\bar{R}^b$	Λ	μ^c	e	h	m_e	F
α^{-1}	0.676	−0.399	0.142	−0.010	−0.058	0.725	−1.086	−1.495	−0.142	−0.943
K^a	*−0.186*	6.808	−13.206	−0.005	3.470	−0.428	7.203	14.006	13.207	−6.003
N_A	*0.034*	*−0.983*	26.516	−0.052	−6.948	0.153	−13.400	−26.658	−26.373	13.116
$\bar{R}^b$	*−0.064*	*−0.011*	*−0.053*	0.036	0.014	−0.011	0.041	0.072	0.052	−0.011
Λ	*−0.013*	*0.249*	*−0.253*	*0.014*	28.544	−0.062	3.542	7.027	6.911	−3.406
μ^c	*0.388*	*−0.072*	*0.013*	*−0.025*	*−0.005*	5.165	−1.165	−1.604	−0.153	−1.012
e	*−0.457*	*0.956*	*−0.902*	*0.075*	*0.230*	*−0.178*	8.330	15.573	13.401	−5.071
h	*−0.334*	*0.986*	*−0.951*	*0.070*	*0.242*	*−0.130*	*0.991*	29.651	26.661	−11.085
m_e	*−0.034*	*0.986*	*−0.997*	*0.053*	*0.252*	*−0.013*	*0.904*	*0.953*	26.376	−12.972
F	*−0.404*	*−0.811*	*0.898*	*−0.020*	*−0.225*	*−0.157*	*−0.619*	*−0.718*	*−0.890*	8.045

[a] $K \equiv A_{Bi69}/A$. [b] $\bar{R} \equiv \Omega_{Bi69}/\Omega$. [c] $\mu \equiv \mu_\mu/\mu_p$.

where the v_{ij} are to be expressed, for example, in (ppm)^2. Equation (33.4) may also be written in terms of correlation coefficients defined by $r_{ij} = v_{ij}/(v_{ii}v_{jj})^{1/2} = v_{ij}/\epsilon_i\epsilon_j$ (note that $r_{ii} \equiv 1$):

$$\epsilon_k^2 = \sum_{i=1}^{N} \left(\frac{\partial Q_k}{\partial x_i}\right)^2 \epsilon_i^2 + \sum_{i \neq j}^{N} r_{ij}\epsilon_i\epsilon_j \frac{\partial Q_k}{\partial x_i}\frac{\partial Q_k}{\partial x_j}. \quad (33.7)$$

Similarly for eq (33.6):

$$\epsilon_k^2 = \sum_{i=1}^{J} Y_{ki}^2\epsilon_i^2 + \sum_{i \neq j}^{J} r_{ij}\epsilon_i\epsilon_j Y_{ki}Y_{kj}, \quad (33.8)$$

where the ϵ_i are to be expressed in ppm. Clearly, if $r_{ij} = 0$ for $i \neq j$ (i.e., no correlation), then eqs (33.7) and (33.8) reduce to the usual law of error propagation for uncorrelated quantities.

Table 33.4 gives the combined variance-covariance and correlation coefficient matrix for our recommended set of constants. For the convenience of the reader, we have expanded this matrix to include e, h, m_e, and F in addition to the constants actually used as unknowns in the adjustment. Such an expansion follows from the fact that the covariance of two quantities Q_k and Q_s is simply

$$v_{ks} = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial Q_k}{\partial x_i}\frac{\partial Q_s}{\partial x_j} v_{ij}. \quad (33.9)$$

If Q_k and Q_s are of the form given in eq (33.5), we can then write in place of eq (33.9)

$$v_{ks} = \sum_{i=1}^{J}\sum_{j=1}^{J} Y_{ki}Y_{sj}v_{ij}, \quad (33.10)$$

where the v_{ij} are to be expressed in (ppm)^2.

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As an example of the use of these matrices, we compute the ratio h/e and its uncertainty. Combining the two equations for h and e given earlier in this section yields

$$h/e = [2/(2e/h)_{\mathrm{B169}}]\cdot K\bar{R}\,. \qquad (33.11)$$

Taking $(2e/h)_{\mathrm{B169}}$, K, and $\bar{R}$ as given in table 33.2, we then obtain $h/e = 4.135701 \times 10^{-15}$ J·s·C^{-1}. To calculate the uncertainty in h/e we use eq (33.6) and table 33.4. Letting K correspond to $j=2$ and $\bar{R}$ to $j=4$ gives

$$\epsilon^2_{h/e} = Y_2^2 v_{22} + 2Y_2Y_4v_{24} + Y_4^2 v_{44}. \qquad (33.12)$$

(Note that auxiliary constants are always assumed to be exactly known.) Comparing eq (33.11) with eq (33.5) yields $Y_2 = 1$ and $Y_4 = 1$. Thus we obtain from eq (33.12) and table 33.4,

$$\epsilon^2_{h/e} = [6.808 - 2(0.005) + 0.036]\,(\mathrm{ppm})^2, \qquad (33.13)$$

or $\epsilon_{h/e} = 2.614$ ppm. An alternate procedure would be to evaluate $\epsilon_{h/e}$ directly from table 33.4; then e corresponds to $j = 7$ and h to $j = 8$, and we find

$$\epsilon^2_{h/e} = Y_7^2 v_{77} + 2Y_7Y_8v_{78} + Y_8^2 v_{88}$$

$$= [8.330 - 2(15.573) + 29.651]\,(\mathrm{ppm})^2, \qquad (33.14)$$

which of course also yields $\epsilon_{h/e} = 2.614$ ppm.

V. Conclusions

Here we summarize the main features of the present work as well as attempt to put it in perspective with regard to similar past efforts.

A. Comparison with Past Adjustments and Overall Quality of Present Adjustment

In the following two sections, we compare selected values of our best WQED and final recommended constants with their appropriate counterparts resulting from the two most recent adjustments; and point out what we consider to be the present major areas of difficulty in the fundamental constants field and the future research necessary to eliminate those difficulties.

34. Changes in the Values of Selected Constants

In table 34.1 we compare our 1973 WQED values for several constants with the similar WQED values given by Taylor et al. [0.1] in their 1969 adjustment. From the table, it is clear that the changes in α^{-1}, e, h, m_e, and Λ are well within the respective one standard deviation uncertainties of the 1969 results. However, this is obviously not the case for N_A, μ_p'/μ_N, and F. These quantities have changed three to four times their respective 1969 uncertainties. The reason for this, of course, is that in their 1969 adjustment, Taylor et al. discarded the so called 'high values" of μ_p'/μ_N, retaining the "low values" and the Craig et al. determination of the Faraday which were highly compatible (see sec. III. A.29). In the present work, we have deleted this determination of the Faraday and also that of Marinenko and Taylor (which is in good agreement with that of Craig et al.), and have used for μ_p'/μ_N the two recent sub-ppm determinations of Mamyrin et al. and of Petley and Morris which, although in excellent agreement, are some 10 to 30 ppm larger than the low values used by Taylor et al. in 1969. It should also be noted that any other quantity which

TABLE 34.1. A Comparison of our best WQED values for α^{-1}, e, h, m_e, N_A, μ_p'/μ_N, F, and Λ with the WQED values resulting from the 1969 adjustment of Taylor et al.[a]

Quantity	Units	Value, this adjustment	Uncertainty (ppm)	Value, 1969 adjustment	Uncertainty (ppm)	Change 1973–1969 (ppm)
α^{-1}		137.03612(15)	1.1	137.03608(26)	1.9	+0.3
e	10^{-19} C	1.6021876(50)	3.1	1.6021901(81)	5.0	−1.6
h	10^{-34} J·s	6.626167(38)	5.7	6.626186(57)	8.5	−2.9
m_e	10^{-31} kg	9.109533(47)	5.1	9.109553(56)	6.2	−2.2
N_A	10^{23} mol^{-1}	6.022046(31)	5.2	6.022174(41)	6.8	−21
μ_p'/μ_N		2.7927740(11)	0.38	2.792709(17)	6.2	+23
F	10^4 C·mol^{-1}	9.648447(29)	3.0	9.648667(54)	5.6	−23
Λ		1.0020771(54)	5.3	1.0020762(53)	5.3	+0.9

[a] Ref. [0.1].

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TABLE 34.2. A Comparison of our final recommended values for α^{-1}, e, h, m_e, N_A, μ_p'/μ_N, and F with those final recommended values resulting from the 1969 adjustment of Taylor et al., and the 1963 adjustment of Cohen and DuMond[a]

Quantity[b]	Value, this adjustment, and ppm uncertainty		Value, 1969 adjustment, and ppm uncertainty		Change 1973–1969 (ppm)	Value, 1963 adjustment, and ppm uncertainty		Change 1973–1963 (ppm)
α^{-1}	137.03604(11)	0.82	137.03602(21)	1.5	+0.15	137.0388(6)	4.4	−20
e	1.6021892(46)	2.9	1.6021917(70)	4.4	−1.6	1.60210(2)	12	+56
h	6.626176(36)	5.4	6.626196(50)	7.6	−3.0	6.62559(16)	24	+88
m_e	9.109534(47)	5.1	9.109558(54)	6.0	−2.6	9.10908(13)	14	+50
N_A	6.022045(31)	5.0	6.022169(40)	6.6	−21	6.02252(9)	15	−79
μ_p'/μ_N	2.7927740(11)	0.38	2.792709(17)	6.2	+23	2.79268(2)	7.2	+34
F	9.648456(27)	2.8	9.640670(54)	5.5	−22	9.64870(5)	5.2	−25

[a] Refs. [0.1] and [29.1].
[b] The units for e are 10^{-19} C; for h, 10^{-34} J·s; for m_e, 10^{-31} kg; for N_A, 10^{23} mol^{-1}; and for F, 10^4 C·mol^{-1}.

depends on N_A (such as m_p), or on μ_p'/μ_N (such as m_p/m_e), will exhibit a similar large change between its 1969 and 1973 values. The dependence of the adjusted values of N_A on μ_p'/μ_N may be seen by expressing μ_p'/μ_N in terms of the variables of our least-squares adjustment. We find from table 29.1

$$(\mu_p'/\mu_N)N_A = \frac{\mu_0 M_B c^2(\mu_p'/\mu_B)[2e/h)_{BI69}]^2}{16R_\infty} \cdot \frac{\alpha}{K^2(\Omega_{BI69}/\Omega)^2}. \tag{34.1}$$

Since none of the quantities on the right side of this equation have changed greatly since 1969 (although there have been significant improvements in accuracy), any change in μ_p'/μ_N between the two adjustments is reflected as a corresponding inverse change in N_A. We also note that the uncertainty in N_A is determined primarily by the uncertainty in K^2.

In table 34.2 we compare our 1973 final recommended values for several constants with the similar recommended values given by Taylor et al. in their 1969 adjustment, and for historical purposes, with those given by Cohen and DuMond in their 1963 adjustment [29.1]. Clearly, the previous statements made regarding table 34.1 apply here as well. We also note that the uncertainties for the 1973 values are lower than the corresponding uncertainties for the 1969 values. There are four main reasons for this.

(a) Four values of γ_p'(low) were used in the present adjustment rather than two as in 1969: Those of ETL, NBS, NPL, and VNIIM, vs those of NBS and NPL. Although we expanded the *a priori* uncertainties assigned the four by the multiplicative factor 1.43, the uncertainty in their weighted mean is still less than it was for the weighted mean of the NBS and NPL results used in 1969. This is due in part to the increased number of values in 1973 as well as to their lower individual uncertainties.

(b) The Josephson effect values of $2e/h$ available in 1973 (in as-maintained electrical units) are so precise that $2e/h$ may be taken to be an auxiliary constant. This was not the case in 1969. At that time, the 2.4 ppm uncertainty assigned the only available Josephson effect measurement of $2e/h$ made it necessary to include it as a stochastic input datum. The net effect of (a) and (b) together is to reduce the uncertainty in α_{WQED} from 1.9 ppm in 1969 to 1.1 ppm in 1973 (see table 34.1).

(c) Many more items of QED data have been included in our 1973 adjustment than were included in the 1969 adjustment of Taylor et al. Indeed, they only used the value of α derived from the hydrogen hyperfine splitting. Although we have expanded the *a priori* uncertainties assigned the QED data in the present work by the multiplicative factor 1.40, the QED data still provide a value of α with an uncertainty of about 1.2 ppm. This is significantly less than the 2.6 ppm uncertainty of the Hhfs value of α used by Taylor et al. in 1969, and is due in part to the many more items of QED input data used here and to the improvement in the uncertainty assignment of α(Hhfs) from 2.6 ppm to 1.6 ppm.

(d) The two values of μ_p'/μ_N which we have used in the 1973 adjustment have uncertainties between 13 and 60 times less than the uncertainties of the values used by Taylor et al. This accounts in large measure for the decrease in the uncertainties in N_A and F between the 1969 and 1973 adjustments.

Tables 34.1 and 34.2 once again emphasize the point made in ref. [0.1]: ". . . no set of fundamental constants should be taken as Gospel truth." Although we feel that the present adjustment brings us another step closer to that truth, we also ". . . . recognize that further significant changes in our knowledge of the constants may well take place."

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35. Current Problem Areas and Future Research

We believe that the present state of our knowledge concerning the fundamental physical constants, while satisfactory in some cases, is extremely unsatisfactory in others.

Perhaps the area of greatest concern is the inconsistency between the various determinations of the gyromagnetic ratio of the proton. The 10 ppm spread in the four presently available low field measurements is disturbing. Although the χ^2 for the distribution of the four determinations is not statistically improbable (odds approximately 10:1 against), the large scatter forced us to expand the *a priori* uncertainties of the low-field measurements by the factor 1.43 and prevented the use of α_{WQED} as a critical test of its QED counterpart.

Some resolution of this situation might be reached if increased accuracy could be achieved in the high field determinations of γ_p' and in the determination of the ampere conversion factor, K. With the presently available data the uncertainty (standard deviation) of the low field γ_p' determinations is 2.3 ppm, that of the high field determinations is 6.8 ppm. If the accuracy of the high field measurements could be improved by a factor of 3 so as to bring it equal to that of the low field data, we would not only have a value of K accurate to 1.6 ppm [see eq (14.12)], but also a significantly improved value for the Avogadro constant [see eq (34.1)] and independent verification of the Faraday [see eq (29.4)], as has been recently emphasized by Taylor [35.1].

We thus conclude that improvements in the accuracy of measurements of the gyromagnetic ratio by both the low field and high field techniques (hopefully, to the few parts in 10^7 level) should be considered of the highest priority in the area of precision measurements; until this is achieved, we are severely limited as to what can be said experimentally about the existence of proton polarizability, the completeness of the theory of the muon hyperfine splitting or even a possible critical test of Wyler's intriguing theory of the fine-structure constant [35.2].[35] It is therefore fortunate that several groups are working on this problem [14.13, 14.19, 35.4].

Alternatively, a direct remeasurement of the Faraday constant would help to resolve the question of possible systematic error in the existing measurements of that quantity. The two currently available Faraday constant determinations are so inconsistent with the other data that they had to be deleted. This is, of course, a highly disturbing situation. Although the other data appear to be sufficiently reliable and consistent that the finger of suspicion points unequivocally at the Faraday, it would be more satisfying if a direct experimental confirmation of the incorrectness of the two existing values was obtained. The Faraday determinations now underway at NBS and NPL will hopefully resolve this question [13.4, 35.5, 35.6].

The unsatisfactory situation with respect to the x-ray data which faced Taylor et al. in 1969 is still present today. The only two new items of data which have become available since then are A. Henins' 10 ppm determination of the ratio Λ, and the 33 ppm determination by van Assche et al. of the electron Compton wavelength λ_C. Unfortunately, none of the combined x-ray-optical interferometer experiments now underway [16.10, 35.7, 35.8] have yielded a result sufficiently reliable to include in an adjustment (see secs. II. B.16 and 17). We trust that this situation will not continue indefinitely and that x-ray measurements will play an important role in future adjustments, especially in determining a value of the Avogadro constant and in resolving the Faraday discrepancy once and for all [see eq (29.4)].

While the situation with the QED data has improved considerably since 1969, there are still some major problems. These include: (1) The extremely discrepant nature of the supposedly highly accurate Kaufman, Lamb et al. determination of $(\Delta E - S)_H$, implying perhaps that the kind of experiments used to determine hydrogen fine-structure are not as well understood as believed. Another indication of this possibility is the apparent magnetic field dependence of the hydrogen fine-structure measurements [23.12] and their generally low implied values of α^{-1}. (2) The uncertainty in the theoretical expression for the hydrogen hyperfine splitting due to our lack of complete knowledge concerning the proton polarizability; (3) the relatively large uncertainty in the presently available determinations of μ_μ/μ_p which limits the accuracy of the value of α which may be derived from the combined Chicago-Yale 0.4 ppm determination of the muonium hyperfine splitting, ν_{Mhfs}; and (4) the uncertainty in the theoretical expressions for the hyperfine splitting in muonium and positronium and for the fine-structure in helium. The fact that we had to expand the *a priori* uncertainties assigned the QED data by the multiplicative factor 1.40 in order to make them more compatible quantitatively reflects some of the problems with these data. We strongly urge that work aimed at eliminating these difficulties be carried out at the earliest possible time. The theory of the proton polarizability and of the hyperfine splitting in muonium and positronium would seem to be of particular importance.

We also note that if the experiments referred to above are successful in determining γ_p'(low) with an accuracy of few parts in 10^7, then improved measurements of R_∞, Ω_{B169}/Ω, and μ_p'/μ_B (e.g., accuracies of a few parts in 10^8) would be useful in order to obtain a value of the fine-structure constant with the highest possible accuracy [see eq (31.1)].

[35] Wyler's theoretical value for α^{-1} is $(16\pi^4/9)(120/\pi^5)^{1/4} = 137.0360824\ldots$, which may be compared with our final recommended value, 137.03604(11). Wyler also predicted [35.3] $m_p/m_e = 6\pi^5 = 1836.118108\ldots$, which may be compared with our final recommended value, 1836.15152(70).

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While the Newtonian gravitational constant, the gas constant, and the Stefan-Boltzmann constant play no role as yet in a least-squares adjustment, they are still of great intrinsic importance. In view of the fact that no postwar measurements of R exist, it would seem that new and improved measurements would be especially in order. Thus, Quinn's [25.6] recent proposal for determining R from velocity of sound measurements should no doubt be actively pursued.

In conclusion, we believe that there is much useful work yet to be done in the fundamental constants field and that the romance of the next decimal place should be passionately pursued, not as an end in itself but for the new physics and deeper understanding of nature that presently lie concealed there.

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Notes Added in Proof

We note here recent developments relevant to the subject matter of this paper that have occurred or have come to our attention since its completion. (Each paragraph is keyed to the corresponding section in the text.)

1. *2e/h from the Josephson Effect*. Experimental tests of the exactness of the Josephson frequency-voltage relation (upon which the determination of $2e/h$ depends) have yet to uncover any deviations. Two of the more recent attempts to detect such deviations are those of J. C. Gallop (National Physical Laboratory Report Qu 25, August, 1973) and J. C. Macfarlane [Appl. Phys. Lett. **22**, 549 (1973)]. T. A. Fulton [Phys. Rev. B **7**, 981 (1973)] has shown theoretically that any "corrections" to $2e/h$ as obtained from the Josephson frequency-voltage relation would imply a breakdown in Faraday's law.

2,4. *Volt and Ohm Intercomparisons*. A regular triennial international comparison at BIPM of the as-maintained units of voltage and resistance of the various national laboratories was carried out during the first half of 1973. However, at the time of this writing (November, 1973), the final results of these intercomparisons were not yet available.

3. *Speed of Light*. T. G. Blaney et al. [Nature **244**, 504 (1973)] have recently confirmed one of the more important intermediate frequency ratios of Evenson et al.'s [3.1] measurement of the frequency of the methane stabilized He-Ne laser which determined the value of c recommended by the CCDM at their June, 1973, meeting (see sec. II.A.3). We have, of course, adopted the CCDM value as our recommended value. We also note that the Comité International des Poids et Mesures (CIPM), at its 62nd meeting held in October, 1973, has now approved the June recommendations of the CCDM (E. Ambler, private communication).

7. *Bound State g-Factor Corrections*. The theoretical bound state g-factor corrections of Grotch and Hegstrom which were used in section II.A.7 have received additional experimental support. J. S. Tiedeman and H. G. Robinson (*Atomic Physics 3*, Ed. by S. J. Smith and G. K. Walter (Plenum Press, New York, 1973), p. 85) report the preliminary experimental result

$$g_j(\mathrm{H},1^2S_{1/2})/g_e = 1-17.69(10) \times 10^{-6},$$

which compares favorably with the theoretical result of eq (7.2a), $1-17.705 \times 10^{-6}$. Furthermore, Grotch and Hegstrom [Phys. Rev. A **8**, 1166 (1973)] have extended their work to helium and find

$$g_j(\mathrm{He},2\,^3S_1)/g_j(\mathrm{H},1\,^2S_{1/2}) = 1-23.212 \times 10^{-6}.$$

(See also M. L. Lewis and V. W. Hughes, Phys. Rev. A **8**, 2845 (1973).) This result is in good agreement with the experimental measurements of E. Aygün, B. D. Zak, and H. A. Sugart [Phys. Rev. Lett. **31**, 803 (1973)] who find the ratio to be $1-23.50(30) \times 10^{-6}$.

9. *Atomic Masses*. The relative atomic masses of the nuclides of Wapstra, Gove, and Bos which are listed in table 9.1 and which we have used herein have been further updated by these workers prior to final publication by taking into account the most recent data. However, the resulting changes in the values of table 9.1 (one or two digits in the last place) are entirely negligible as far as our recommended values are concerned.

10. *Rydberg Constant*. We note here that laser saturated absorption spectroscopy may shortly yield a value of R_∞ accurate to 1 or 2 parts in 10^8. (See T. W. Hänsch, I. S. Shahin, and A. L. Schawlow, Nature Phys. Sci. **235**, 63 (1972).) Indeed, Hänsch (private communication) is well on the way towards obtaining such a result.

12. *Absolute Ampere*. In a private communication, S. V. Gorbatsevich has provided us with further results of the VNIIM absolute ampere experiments. However, a detailed description of the work is not given. He reports that $A_{\mathrm{VNIIM},67}/A$ was found to be as follows in the years indicated:

1966: 1.0000165(27) (2.7 ppm),

1968: 1.0000158(16) (1.6 ppm),

1969: 1.0000162(18) (1.8 ppm).

The quoted uncertainties are the statistical standard deviations of the means of some 80 to 90 measurements. Correcting for known changes in the as-maintained VNIIM ohm, and using the results of the 1967 BIPM triennial intercomparison, Gorbatsevich finds for $K \equiv A_{\mathrm{BI69}}/A$:

1966: 0.9999965(27) (2.7 ppm),

1968: 0.9999963(16) (1.6 ppm),

1969: 0.9999973(18) (1.8 ppm).

These data yield a weighted mean of 0.9999967(12). Taking into account the most recent determinations of the gravitational acceleration introduces a -1.0 ppm correction to this result while the estimated effect of wire strain leads to a (2.0 ± 1.0) ppm correction. Thus, Gorbatsevich reports the final value

$$K = 0.9999977(60)\ (6.0\ \mathrm{ppm}),$$

where the quoted uncertainty now includes both random and systematic uncertainty components.

The above result may be compared with our recommended value, $K = 1.0000007(26)$ (2.6 ppm). The 3.0

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ppm difference between the two is clearly consistent with the assigned uncertainties. Although information concerning the VNIIM result sufficient for us to seriously consider it as a potential stochastic input datum is presently not available, we do note that if it were included, our recommended values for the various constants would change by only small fractions of their assigned uncertainties. Similarly, the uncertainties themselves would change by only small amounts. This may be readily seen from table I. We give there in the column labeled Case A representative results of an adjustment identical to that used to obtain our final recommended values but with the VNIIM datum included. (For this adjustment, χ^2 is 14.71 for 22 degrees of freedom; $R_B = 0.82$.) For reference purposes, we also repeat in the second column of the table the final recommended uncertainties of the relevant quantities as originally given in tables 33.1 and 33.2. A comparison of the two columns clearly shows that any changes in our recommended values due to the VNIIM result would be entirely negligible.

15. *Proton Magnetic Moment in Nuclear Magnetons.* The final report by Petley and Morris [J. Phys. A: Math., Nucl. Gen. **7**, 167 (1974)] on their omegatron measurements of μ_p'/μ_N gives

$$\mu_p'/\mu_N = 2.7297748(20)\ (0.72\ \text{ppm}),$$

as the final result of their work. This may be compared with the value given in their preliminary report, ref. [15.13], and which we have used in the present paper:

$$\mu_p'/\mu_N = 2.7297748(23)\ (0.82\ \text{ppm}).$$

Although the result itself is unchanged, the final uncertainty is 0.1 ppm less than the preliminary uncertainty. However, this change is obviously entirely negligible as far as our final recommended values are concerned. Its primary effect would be to lower the uncertainty of our recommended values for μ_p'/μ_N and other closely related quantities such as μ_p/μ_N and m_p/m_e from 0.38 ppm to 0.37 ppm. With regard to the numerical values of these three constants themselves, they would increase by only 0.02 ppm. Other constants would generally change by less than 0.01 ppm.

16. *Ratio, kxu to ångström.* The combined x-ray and optical interferometer measurement of Λ by Deslattes and collaborators at the National Bureau of Standards which was discussed in sections II.B.16 and 17 has advanced to the point where reliable results are now available. R. D. Deslattes and A. Henins [Phys. Rev. Lett. **31**, 972 (1973)] report that

$$\Lambda = 1.0020802(10)\ (1.0\ \text{ppm}),$$

based on the x-unit scale we have used herein ($\lambda(\text{Cu}K\alpha_1) \equiv 1.537400$ kxu). This value exceeds our final recommended value given in table 33.2,

$$\Lambda = 1.0020772(54)\ (5.3\ \text{ppm}),$$

by only 3.0 ppm, well within the 5.3 ppm uncertainty assigned the latter. Thus, the two values are quite consistent.

It is of interest to investigate the effect of this new result on our recommended values of the constants. If it were simply used as an additional stochastic input

TABLE I. Possible changes in our final recommended values for various changes in input data

Quantity	ppm uncertainty in final recommended value (from tables 33.1, 33.2)	ppm change in recommended value (first number), and in its uncertainty (second number)							
		Case A[a]		Case B[b]		Case C[c]		Case D[d]	
α^{-1}	0.82	0.03	0.00	−0.01	0.00	0.01	0.00	−0.03	0.04
A_{B169}/A	2.6	−0.5	−0.2	0.3	−0.1	0.2	−0.1	0.0	0.0
N_A	5.1	0.9	−0.4	−0.7	−0.1	−0.5	−0.2	0.0	0.0
Ω_{B169}/Ω	0.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Λ	5.3	−0.2	0.0	2.9	−4.3	3.1	−4.3	0.0	0.0
μ_μ/μ_p	2.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
e	2.9	−0.5	−0.2	0.4	−0.1	0.2	−0.1	0.0	0.0
h	5.4	−1.0	−0.4	0.7	−0.1	0.5	−0.2	0.1	0.0
m_e	5.1	−0.9	−0.4	0.7	−0.1	0.5	−0.2	0.0	0.0
F	2.8	0.4	−0.1	−0.3	0.0	−0.3	−0.1	0.0	0.1
γ_p'	2.8	0.4	−0.1	−0.3	−0.1	−0.3	−0.1	0.0	0.0
μ_p'/μ_N	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

[a] VNIIM value for $K \equiv A_{B169}/A$ included. [b] Deslattes and Henins' value for Λ included. [c] Deslattes-Henins value for Λ included but most x-ray data deleted. [d] Carroll and Yao's calculation for C_4 used.

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datum with an uncertainty as given, then we would obtain the results summarized in table I in the column labeled Case B. Clearly, our final recommended values for the various constants would change by only small fractions of their assigned uncertainties and the uncertainties themselves would change by only small amounts. The only exception, of course, would be Λ itself. For this quantity we would find

$$\Lambda = 1.00208010(98)\ (0.98\ \text{ppm}),$$

very nearly the Deslattes-Henins input value. (For this adjustment, χ^2 is 14.81 for 22 degrees of freedom; $R_B = 0.82$.)

The above procedure would, however, be rather unrealistic since Deslattes and Henins' result now makes obsolete all of the x-ray data except perhaps the two measurements of $N_A\Lambda^3$. (This is obviously true for the three values of Λ, eqs (16.3), (16.5), and (16.7). It is also true for the two values of λ_C, eqs (18.2) and (18.3), since λ_C in metres as determined from R_∞ and α is so well known that the x-ray measurements of λ_C are essentially determinations of Λ.) Thus, using the same data as were used in our final adjustment but with eqs (16.3), (16.5), (16.7), (18.2), and (18.3) replaced by the Deslattes-Henins value of Λ, and without expanding the uncertainties of either this value or the two values of $N_A\Lambda^3$ since they are highly compatible, we obtain the results of Case C in table I. (For this adjustment, χ^2 is 11.21 for 18 degrees of freedom; $R_B = 0.79$.) Clearly, the remarks made concerning Case B hold for this case as well. The adjusted value of Λ would be

$$\Lambda = 1.00208027(98)\ (0.98\ \text{ppm}).$$

We also note that with the availability of the Deslattes-Henins result, including even the two $N_A\Lambda^3$ determinations as was done above is questionable. The reason is that the new result, in combination with the two measurements of $N_A\Lambda^3$, yields a value of N_A with an uncertainty just about three times larger than the uncertainty in the value of N_A implied by the other data [see eq (34.1)]. Conversely, the value of Λ implied by this value of N_A and the two available determinations of $N_A\Lambda^3$ has an uncertainty close to five times that of the Deslattes-Henins result. Thus, one could seriously consider discarding all of the x-ray data except the new value of Λ.

19. *Electron Anomalous Moment*. Two new values of C_4 have become available since the completion of our paper. The recalculation by Wright and Levine discussed in section II.C.19 has now been completed [M. J. Levine and J. Wright, Phys. Rev. D, **8**, 3171 (1973)]. They report

$$\text{Levine, Wright: } C_4 = 0.883(60).$$

This result includes the analytic (exact) values for a number of graphs as calculated by M. J. Levine and R. Roskies [Phys. Rev. Lett. **30**, 772 (1973)], some of which have been recently confirmed by K. A. Milton, W. Tsai, and L. L. DeRaad (Phys. Rev., to be published). A new, completely numerical calculation by R. Carroll and Y. P. Yao [Phys. Lett. **48B**, 125 (1974)] using the mass operator formalism gives

$$\text{Carroll, Yao: } C_4 = 0.737(60).$$

Both of these new results may be compared with that of Kinoshita and Cvitanovic which we have used in the present work [eq (19.6)]:

$$\text{Kinoshita, Cvitanovic: } C_4 = 1.024(40). \qquad (19.6)$$

Clearly, these three values of C_4 are in rather poor agreement. The value of χ^2 for their weighted mean is 16.43 ($R_B = 2.9$). The probability for two degrees of freedom that a value of χ^2 this large or larger could occur by chance is less than 3 in 10^4. Further work will be required to resolve this discrepancy.

Two additional calculations of C_3 have also recently been completed. J. Calmet and A. Peterman [Phys. Lett. **47B**, 369 (1973)] find $C_3 = 0.366(10)$; C. Chang and M. J. Levine (as quoted in the above Levine and Wright paper) find $C_3 = 0.370(13)$. These two results are in excellent agreement and yield a weighted mean of $C_3 = 0.367(8)$. The value of C_3 we have used in the present paper as obtained by Aldins et al., $C_3 = 0.36(4)$ [see eq (19.5)], is obviously in quite good agreement with the two new calculations.

It is of interest to investigate the effect of the new results for C on our final recommended values. (The effect of the two new calculations of C_3 alone would be undiscernible since a 0.007 change in C_3 and hence in C corresponds to only a 0.08 ppm change in the implied value of $\alpha^{-1}(a_e)$.) Following section II.C.19 but taking the value of C_4 as appropriate and $C_3 = 0.367(8)$ as above, we find

$$\begin{gathered}\text{Levine, Wright}\\ C = 1.152(61),\\ \alpha^{-1}(a_e) = 137.03543(42)\ (3.1\ \text{ppm});\end{gathered}$$

$$\begin{gathered}\text{Carroll, Yao}\\ C = 1.006(56),\\ \alpha^{-1}(a_e) = 137.03521(42)\ (3.1\ \text{ppm}).\end{gathered}$$

These values may be compared with the corresponding values used in the present work which were based on the results of Kinoshita and Cvitanovic, and the value $C_3 = 0.36(4)$ of Aldins et al.:

$$C = 1.285(57), \qquad (19.7)$$

$$\alpha^{-1}(a_e) = 137.03563(42)\ (3.1\ \text{ppm}). \qquad (19.8)$$

The uncertainty in these values of $\alpha^{-1}(a_e)$ is due

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primarily to the 3.0 ppm experimental uncertainty in a_e [eq(19.1)]. Thus, this uncertainty masks the changes in $\alpha^{-1}(a_e)$ of (-1.5 ± 0.8) ppm and (-3.1 ± 0.8) ppm due to the changes in C which follow respectively from the Levine-Wright and Carroll-Yao calculations.

As an example of the influence of these new values of $\alpha^{-1}(a_e)$ on our final recommended values for the constants, we consider the more extreme case, that of Carroll and Yao. Using the value of $\alpha^{-1}(a_e)$ implied by their calculation in place of that implied by the work of Kinoshita and Cvitanovic, eq (19.8), we first find that the QED data are somewhat more incompatible than previously; the multiplicative factor to obtain compatibility (i.e., R_B) is 1.56 compared with 1.40 (see sec. III.C.31). Then, using the Carroll-Yao $\alpha^{-1}(a_e)$ result and applying this expansion factor to all of the QED data but otherwise repeating the adjustment used to obtain our recommended values, we find the results of Case D in table I. Once again we see only occasional very minor changes in both the numerical values of our recommended constants and in their uncertainties.

We also note that a new determination of a_e by F. L. Walls and T. S. Stein has recently been reported [Phys. Rev. Lett. **31**, 975 (1973)]. Using a bolometric technique to observe the g-2 resonance of a stored electron gas, they find $a_e = 0.001159667(24)$ (21 ppm), in good agreement with the result of Wesley and Rich, eq (19.1), but the uncertainty of the Walls-Stein value is some 7 times larger.

21. *Muon Magnetic Moment*. The 7.8 ppm correction due to Jarecki and Herman which was used in section II.C.21 to take into account the pressure shift in $g_j(M)$ seems to be slightly in error. These workers apparently used the gage pressure (230 psi) of the two relevant Chicago measurements rather than the absolute pressure. The correction should actually be 8.3 ppm. The net effect of this change is to lower the value of μ_μ/μ_p resulting from the Chicago work, eq (21.5), by 0.5 ppm to

$$\mu_\mu/\mu_p = 3.1833480(148)\ (4.7\ \text{ppm}).$$

Thus, the Chicago result is now in even better agreement with the more accurate value of Crowe, Williams et al., eq (21.1), than it was previously.

It is rather obvious that this approximate one tenth standard deviation shift would have little impact on our recommended values. Its effect on the weighted mean of the three direct measurements of μ_μ/μ_p, eqs (21.1), (21.2), and (21.5), is to decrease it by 0.1 ppm. The quantity $\alpha^{-1}(\nu_{\text{Mhfs}})$, eq (22.4), would thus decrease by only 0.05 ppm, and the consistency factor for the QED data would change by only 0.01 from 1.40 to 1.39. The vast majority of our recommended values would change by no more than 2 or 3 parts in 10^8, with the exception of μ_μ/μ_p and other closely related quantities such as m_μ/m_e and m_μ; these would change by 0.07 ppm. The uncertainties of the recommended values would remain as given in tables 33.1–33.3

22. *Muonium Hyperfine Splitting*. The final report by the Chicago group of their zero-field Ramsey resonance work originally described in ref. [22.14] has now appeared [D. Favart et al., Phys. Rev. A **8**, 1195 (1973)]. However, the results are unchanged and are as given in table 22.2. The Chicago group has also carried out an additional measurement using this same method but with Ar as the stopping gas [H.G.E. Kobrak et al., Phys. Lett. **43B**, 526 (1973)]. They report $\nu_{\text{Mhfs}}(p) = 4463265.3(2.4)$ kHz for a density corresponding to $p = 1802$ torr. But in obtaining this pressure, the real pressure-volume relation for Ar was not taken into account. Using the virial coefficient data of J. H. Dymond and E. B. Smith [*The Virial Coefficients of Gases, a Critical Compilation* (Clarendon Press, Oxford, 1969)], we find $p = 1805$ torr. (It should be noted that this difference is of little practical consequence since the total pressure shift correction is only of order 10 ppm.)

We also take this opportunity to similarly correct the Chicago Ar measurements of Ehrlich et al. [22.13] and to separate their two 12600 torr runs which they had originally combined. (This separation is more consistent with our handling of the Yale data. Note also that all of the Yale data as well as the remaining Chicago data had, in the original papers, been reduced to 0 °C using the true pressure-volume relation for Ar and Kr.) Thus, the Chicago Ar data of table 22.2 now reads

Pressure (torr)	Value (kHz)
3030	4463220(22),
3159	4463249.3(13.2),
7150	4463152.17(2.37),
12734	4463039.7(12.9),
12734	4463020.2(11.4),
1805	4463265.3(2.4),

where we have also slightly revised the uncertainties we had originally assigned the 3150 and 12600 measurements in order to include the systematic effects discussed by Erhlich et al. in ref. [22.13].

Repeating the least-squares fit of section II.C.22 with these and the remaining data of table 22.2, we find

$$\nu_{\text{Mhfs}} = 4463303.82(1.42)\text{kHz}\ (0.32\ \text{ppm}),$$

$$a_{\text{Ar}} = -4.186(65) \times 10^{-9}\ \text{torr}^{-1},$$

$$a_{\text{Kr}} = -10.595(56) \times 10^{-9}\ \text{torr}^{-1},$$

$$b_{\text{Ar}} = 6.27(85) \times 10^{-15}\ \text{torr}^{-2},$$

$$b_{\text{Kr}} = 8.30(1.24) \times 10^{-15}\ \text{torr}^{-2}.$$

(For this fit, χ^2 is 17.56 for 29 degrees of freedom; $R_B = 0.78$.) In comparing these results with those of section II.C.22, eqs (22.2) and (22.3), we see that the changes are extremely minor. The muonium hyperfine

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splitting remains unchanged and its uncertainty is decreased by only 0.08 ppm. Similarly, the various pressure shift coefficients remain essentially unchanged but their uncertainties have decreased.

Since it is necessary to use the theoretical expression for the muonium hyperfine splitting frequency in order to include the experimental value in a least-squares adjustment, and since the 2.0 ppm uncertainty of the former overwhelmingly dominates the several tenths ppm uncertainty of the later, the effect on our recommended values of the above 0.08 ppm decrease in the uncertainty of ν_{Mhfs}(experimental) would be completely undiscernable, i.e., changes of less than 0.01 ppm.

We also note here that the final report on the hyperfine pressure shift measurements of Ensberg and Morgan (ref. [22.16]) for hydrogen isotopes in argon has now appeared [C. L. Morgan and E. S. Ensberg, Phys. Rev. A **7**, 1494 (1973)]. Their final results have changed little from those given in ref. [22.16] and which we have listed in eq (22.1), except that the uncertainties have been considerably reduced. These workers have also accurately measured the temperature dependence of the fractional pressure shift coefficient a [see eq (22.1)] and although its effect would be rather small, for the sake of completeness it should be taken into account in future extrapolations of $\nu_{\mathrm{Mhfs}}(p)$ to zero pressure. This would require full knowledge of the actual temperatures at which the various measurements of $\nu_{\mathrm{Mhfs}}(p)$ were carried out.

22. *Hydrogen Hyperfine Splitting.* Essen et al. in a recent publication [Metrologia **9**, 128 (1973)] have further described their hydrogen maser work at the National Physical Laboratory. They have now taken into account the so called "stem effect" and find that their earlier result, which was reported in ref. [22.18] and which we have given in eq (22.5c), should be modified to

$$\nu_{\mathrm{Hhfs}} = 1420405751.7662(3)\ \mathrm{Hz}.$$

The respective 0.0005 Hz and 0.0007 Hz reductions in ν_{Hhfs} and its uncertainty are, of course, inconsequential as far as our final recommended values are concerned.

24. *Newtonian Gravitational Constant.* At least three experiments are currently underway to determine the Newtonian gravitational constant to greater accuracy: A collaborative effort between the National Physical Laboratory, the University of Edinburgh, and the Instituto di Geodesia e Geofisica of the University of Trieste [A. H. Cook, Contemp. Phys. **9**, 227 (1968); A. Marussi, Memo. Soc. Astron. Ital. **43**, 823 (1972)]; a collaborative experiment between the National Bureau of Standards (Gaithersburg) and the University of Virginia (G. G. Luther and W. R. Towler, private communication); and a collaborative effort between the National Bureau of Standards (Boulder) and the University of Colorado at the Joint Institute for Laboratory Astrophysics (J. E. Faller and B. Koldewyn, private communication).

Official Adoption. Our recommended set of constants, tables 33.1, 33.2, and 33.3, were approved for international use by the CODATA Task Group on Fundamental Constants and adopted officially by the 8th CODATA General Assembly at its September, 1973, meeting in Stockholm, Sweden. A summary report of the present paper by the Task Group giving our recommended values is published in CODATA Bulletin No. 11, December, 1973.

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