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Single-photon detector characterization using correlated photons: the march from feasibility to metrology

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Abstract. Correlated photons can be used to directly measure the detection efficiency of photon counting detectors without any ties to externally calibrated standards. An overview of the history of this technique is given and the paper reviews how to implement it in a practical lab setting. Some of the sources of uncertainty in the technique and how they can be minimized and quantified are discussed. The intent is to provide the information necessary to encourage the movement of this technique from the metrology lab into the general photon-counting detector community.

1. Introduction

The simultaneous creation of two photons allows the efficiency of photon counting detectors to be measured directly in the photon counting regime without relying on external standards. This provides a unique tool for photon counting detector characterization. This is a particularly valuable tool because conventional calibrations (performed by comparison to a reference standard) are much more complex and difficult for photon counting detectors than for analogue detectors. While the two-photon calibration technique has been understood for some time, it is still not widely used in detector characterization. As quantum information technology progresses, it is becoming increasingly important for manufacturers and users of photon counting detectors to accurately characterize their performance. In this article we give an overview of the standard two-photon detector calibration method, with the intent of increasing awareness of this technique for characterizing photon counting detectors and to point out some of the complexities of photon counting metrology. Hopefully this will spur a more widespread adoption of the technique and advances in this field of metrology.

Spontaneous parametric down-conversion [1–3] (PDC) provides the most convenient source of correlated photons for detector calibration. In the down-conversion process, a nonlinear crystal allows photons from a pump laser to be converted into pairs of photons under the constraints of energy and momentum conservation,

$$\begin{aligned} \omega_{\mathrm{p}} &= \omega_1 + \omega_2 \\ \mathbf{k}_{\mathrm{p}} &= \mathbf{k}_1 + \mathbf{k}_2 \end{aligned} \qquad (1)$$

where ω_{p} and $\mathbf{k}_{\mathrm{p}}$ are the frequency and wave vector of the pump, and ω_i and $\mathbf{k}_i$ $(i=1,2)$ refer to a pair of down-converted output photons (wave vectors are

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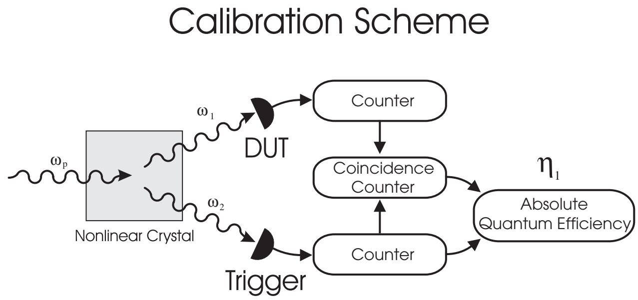

Figure 1. General detection efficiency measurement scheme. Downconverted output photons are shown heading for the DUT and the trigger detector. The determined efficiency, η_1 is the efficiency of the entire detection path from creation of the photon in the crystal to its detection by the detector.

evaluated within the crystal). Because the photons are created in pairs, the detection of one indicates, with absolute certainty, the existence of the other. To measure detection efficiency, the detector under test (DUT) is placed to intercept some of the down-converted photons from the PDC source (see figure 1). A second detector, referred to as the trigger, is positioned to collect a subset of the photons correlated with those seen by the DUT. The phase matching constraints allow the direction, polarization, and spectrum of the correlated photon pairs to be predicted with high certainty, so an optical setup for collecting the photons correlated to those seen by the trigger detector can be chosen in a very direct manner. When the trigger senses a photon (one of a pair), it effectively ‘heralds’ the arrival of the correlated photon at the DUT. The number of counts recorded in a given time by the two detectors and the number of coincidences between channels can be written as

$$\begin{aligned} N_1 &= \eta_1 N \\ N_2 &= \eta_2 N \\ N_{\mathrm{Coinc}} &= \eta_1 \eta_2 N \end{aligned} \tag{2}$$

where η_1 and η_2 are the efficiencies of the two detection channels and N is the number of photon pairs emitted by the crystal during the counting period. It immediately follows that the absolute detection efficiency of channel 1 is simply

$$\eta_1 = \frac{N_{\mathrm{Coinc}}}{N_2}. \tag{3}$$

The remarkable feature of this result is that η_1 is independent of the efficiency of the trigger channel. Thus, we can use imperfect detectors to perform an absolute calibration without any comparison to a reference standard! [3] While this absolute calibration requires no external standard, it is important to note that equation (3) does not give just the quantum efficiency of the DUT alone (η_{DUT}), but rather the detection efficiency of the entire detection channel from where the photon is created within the crystal to where the detection is actually recorded. Any losses within the crystal or in the optical collection system are included in η_1, along with the efficiency of the detector to be measured. This reality must be handled properly to turn this measurement principle into an accurate metrological technique for characterizing detector efficiency.

Another useful feature of PDC-based detector calibration is that the spectral selectivity components for the calibration wavelength do not need to be in the DUT optical path, but can be placed in the trigger path. The constraints in equation (1) can then be used to calculate the range of frequencies that are correlated to the photons incident on the trigger detector. This calculated range can be thought of as a 'virtual' bandpass filter in the DUT path, defining the wavelengths that result in coincidences. This can be of advantage when calibrating a detector in the infrared, where spectral selection may more convenient in the visible trigger channel.

Even though the basic concept behind the PDC calibration method has been known for many years, [3] there is still work to be done improving measurement procedures to increase the precision and reliability, with realistic uncertainty goals of 0.1% or better. These improvements are needed to have a truly useful metrological technique. In the following section we briefly trace the development of two-photon calibration from simple feasibility demonstrations to its implementation as a practical metrological technique. We then review some of the complexities encountered in a practical calibration setup and look at some of the requirements for developing the technique into a rigorous calibration protocol so that it can be used conveniently and with confidence.

2. History

The connection between parametric down-conversion and metrology began early. PDC was predicted in 1961 by Louisell *et al.* [1] and in 1970 [3] the very first experiment to observe coincidences between down-converted photons also included the first detector calibration using a PDC source. In that experiment Burnham and Weinberg measured the detection efficiency of a photomultiplier tube (PMT) using the coincidence method and compared the value with a measurement made using a calibrated lamp source. Although they found a discrepancy of about 30% between the two measurements, they concluded that the two measurements were consistent within their estimated systematic error of $\sim \pm 20\%$. Interestingly, the method was not widely disseminated, as seven years later Klyshko [4, 5] independently proposed that PDC could be used to measure detection efficiency. In 1981 Malygin *et al.* [6] followed up on this proposal and calibrated PMT detectors, although their work was more in the vein of a demonstration effort rather than a work of metrology, as they did not report their uncertainties or give details about any comparison to an independent calibration technique.

In the late 1980s there was an upsurge in interest, and several groups began to look at photon counting detector calibration using PDC. In 1986 Bowman *et al.* [7] demonstrated the PDC calibration technique using avalanche photodiodes (APD) as the photon counting detectors. They reported an uncertainty in their measurement of about 10% but did not make an intercomparison with a conventional standard. In 1987 Rarity *et al.* [8] performed a calibration using APDs, where they validated their results by comparing the PDC results to those obtained using a He–Ne laser attenuated with calibrated neutral density filters. This work was a real advance into the realm of metrology, as they made a careful investigation of the uncertainties in the experiment (the reported uncertainty was $\sim \pm 10\%$). In 1991 Penin and Sergienko [9] reported an earlier calibration of PMT detectors with a statistical error of 3%, but gave no further indication of measurement uncertainties. They also indicated that they compared the results with other reported conventional measurements, but gave no details on how these were obtained. In 1993 Ginzberg *et al.* [10] measured the efficiency of a PMT to an uncertainty of $\pm 10\%$, and no comparison was made to a conventional standard. In 1994 Kwiat *et al.* [11] made a more careful study of the calibration technique, giving a detailed explanation of their method of accounting for uncertainties, and ending with a $\sim 3\%$ uncertainty in the detector efficiency value. In 1995 Migdall *et al.* [12] looked at several more effects. They performed a calibration of a PMT and compared the results to a conventional detector calibrated to a radiometric standard at the National Institute of Standards and Technology (NIST). Multiple comparisons indicated that any measurement bias between the two methods was less than 0.6%, with an estimated uncertainty of $\sim 2\%$ for individual detection efficiency measurements. Five years later, Brida *et al.* [13, 14] performed a careful calibration, with a comparison against a Si trap detector for independent verification. In this work they included the collection optics and the spectral filter losses as part of the DUT (significantly simplifying the comparison of the two measurements), resulting in a 0.5% correlated photon calibration uncertainty and a 1% conventional calibration uncertainty for comparison.

Table 1 summarizes the experimental results showing the general, but uneven trend from demonstration type measurements to more careful metrology efforts. Although the measured values of detection efficiencies have become more precise over time, the uncertainties are still not ideal. In the following section we discuss some of the procedures that can be used to reduce the uncertainties of the PDC calibration technique. Before leaving this history, however, it is worth noting that before the advent of PDC, two-photon detector calibration already had a long history of being carried out using atomic cascade [15–17] sources to produce the correlated photons. While successful, the drawback of this type of source is that the large range of solid angles into which the atom can decay results in low source brightness. In comparison to two–photon cascade, the phase matching constraints of PDC restrict the emission of photon pairs to a narrow range of angles, effectively brightening the two photon source by a factor of $\sim 10^6$. The origin of

Table 1. History of PDC-based calibration/metrology efforts. As best as can be deduced from the literature, these are standard uncertainties ($k = 1$).

Year	First author	Uncertainty	External comparison
1970	Burnham	$\sim 35\%$	Calibrated lamp
1981	Malygin	–	–
1986	Bowman	$\sim 10\%$	–
1987	Rarity	$\sim 10\%$	HeNe + Attenuation
1991	Penin	$> 3\%$	–
1993	Ginzburg	$\sim 10\%$	Published values
1994	Kwiat	$\sim 3\%$	–
1995	Migdall	$< 2\%$	Calibrated Si Detector
2000	Brida	$\sim 0.5\%$	Calibrated Si Detector

detector calibration using coincidence techniques can be traced back even further to the early 1900s, when coincidence techniques were already being suggested and used to study particles emitted in nuclear decay [18–20] and other fundamental processes. Even at this early stage it was clear [21] that the coincidence method is ideally suited to detector metrology.

3. Single-photon detector calibration

There are several sources of uncertainty that arise when calibrating detectors using the photon pairs produced in PDC. The most critical relate to the process of disentangling the efficiency of the detector (η_{DUT}) from the efficiency of the detection channel taken as a whole (η_1). In simple terms, if the trigger detector senses a photon and the DUT does not detect its twin then we do not know whether the photon was actually incident on the detector or was lost upstream in the detection channel. To minimize the uncertainty associated with this, we first design the system to maximize the collection of the photons correlated to those seen by the trigger detector. Then we carefully measure or calculate any residual collection losses that cannot be designed out of the system. The uncertainty of this residual loss contributes, in large part, to the ultimate limit of the uncertainty of the final detector efficiency (η_{DUT}).

Collection system losses can be classified as either conventional *transmittance* losses due to the reflection and absorption or *geometric* losses due to causes such as limiting apertures, finite detector area, or positioning errors. Transmittance losses may be handled straightforwardly; the transmittances of optical components can be measured conventionally with high accuracy, or in some cases the losses can be calculated with good results (e.g. the losses in the down-conversion crystal). In some cases it is even possible to measure the transmission losses *in situ* [22], although this has yet to be demonstrated in a calibration setup. Geometric losses due to positioning errors can also be characterized in a clear-cut manner. For example, the misalignment of the DUT with the centre of the path of photons correlated to those seen by the trigger can be characterized by simply adjusting the detector position.

A more challenging loss to quantify occurs because there is a spread of emission directions and positions for the photons correlated to those seen by the trigger detector. Angular spreading, sometimes referred to as 'entanglement angle' [23], results from a variety of causes. (See [24] for details.) Typically, the most pronounced spreading occurs because the light seen by the trigger detector has a finite spectral bandwidth (this spectral bandwidth is usually due to a combination of spectral filters and angular acceptance). Because different wavelengths of downconversion have different emission angles, the bandwidth seen by the trigger affects the angular spread of the light that must be coupled onto the DUT (figure 2(a)). This requires that the angular acceptance of the DUT be large enough to accept all frequencies correlated to photons arriving at the trigger detector, and also that the frequency-selective elements in the DUT channel be broad enough to transmit these photons.

Angular spreading also occurs because the phase-matching conditions are loosened for crystals of finite length, so that even when the k-vectors for the pump and trigger photons are considered fixed there is a range of possible emission angles for the photon in the DUT channel (figure 2(b)). Additional angular

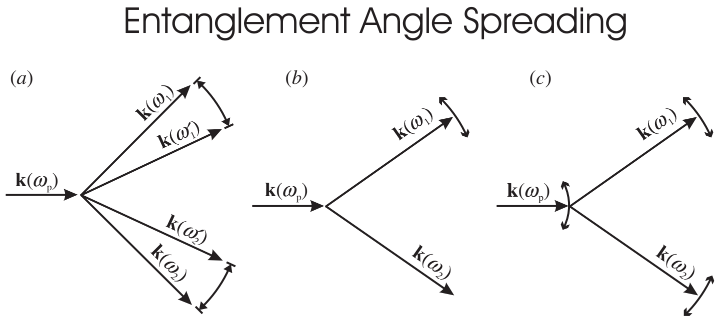

Figure 2. There are several causes for angular spreading in the photons correlated to the trigger: (*a*) different frequencies are emitted in different directions, so when the trigger accepts a range of frequencies and **k**-vectors the DUT must collect the correlated range of frequencies and **k**-vectors; (*b*) phase-matching considerations allow some variation of emission angle even when the pump and trigger **k**-vectors are fixed; (*c*) the pump beam has a range of **k**-vectors due to its finite size.

spreading occurs because there is a range of k-vectors present in the pump beam (due to its finite diameter) so that even when the phase-matching conditions hold perfectly, there is a range of output angles for each frequency (figure 2(c)). In general, a longer down-conversion crystal and a wider pump beam diameter result in a more tightly constrained range of output angles in the PDC output. However, as the pump diameter and crystal length increase, the optics coupling light onto the detector must collect light from a larger range of spatial positions to maintain practical counting rates. Thus it is necessary to balance these effects by carefully matching the pump waist, crystal length, and collection optics to optimize collection efficiency [25].

Regardless of their causes, collection losses must be verified experimentally to ensure that the fraction of correlated photons lost before reaching the DUT is small and can be quantified to the desired level of uncertainty. For example, figure 3 illustrates a test that can be used to verify that the angular acceptance of the DUT collection optics is sufficient. In this instance, the collection angle of the trigger channel is set at 2.2 mrad by placing an iris before its collection lens. Another iris is placed in front of the DUT collection lens, and the detection efficiency of channel 1 is plotted as a function of the acceptance angle defined by the DUT iris. Figure 3 shows the detection efficiency rising and then leveling off with DUT collection angle, while the total signal on the DUT continues to rise. Thus we see that an acceptance angle of 6 mrad was required for the detection efficiency to be determined to within a spread of $\pm 0.25\%$.

There are a number other techniques for quantifying channel losses and other important systematic effects. We detail many of these in [26]. Here we consider the additional example of how detector after-pulsing (i.e. the tendency of a detector to send a false detection signal immediately after a real detection event) can affect efficiency measurements. Figure 4 illustrates this effect. The top histogram records the delays between trigger events and the first DUT signal received after each trigger event. The middle histogram records the delay time for DUT events in cases where the DUT has already fired once for the associated trigger (i.e. every second firing event on the middle histogram has a first firing event recorded on the

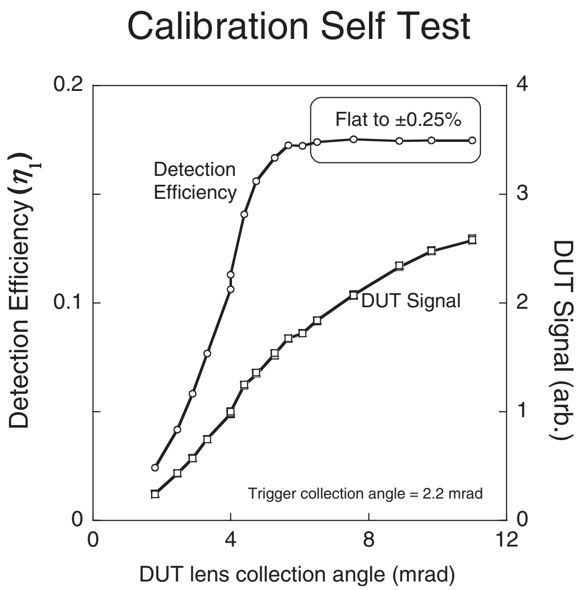

Figure 3. The detection efficiency of channel 1 and its singles count rate are shown as the DUT collection lens iris is varied with the trigger detector collection angle fixed at 2.2 mrad [26].

top histogram), and the bottom histogram shows delays for DUT events where the DUT has already fired twice. The long exponential tail on the top histograms, which is possibly due to non-prompt carrier diffusion in the DUT, presents a problem when determining the detection timing window. If these non-prompt signals are to be included as valid detections, then the coincidence window must be quite wide. However, when the coincidence window is set large enough to include a long tail, it is important to distinguish between first pulses and after-pulses (as is done in figure 4), as the after-pulses should not be included as coincidences. In other words, the system used to count coincidences should add at most one coincidence count per trigger pulse. If the after-pulses were classified as coincidences in a detection system, then the coincidence counts would be over-estimated by about 7% in this example. This particularly presents a problem for conventional calibration, where arrival times may not be recorded.

These examples, along with those discussed in [26], give a flavour for the kinds of things that must be considered in turning this two-photon technique into metrology. In addition to these effects, it is also important to consider other factors such as threshold setting, dead time, nonlinearities (both reversible and permanent), spatial responsivity nonuniformity of the detector, and other timing related issues.

4. Conclusions

The coincidence method for calibrating a photon-counting detector using a PDC source has many advantages over a traditional calibration technique using reference standards. The resulting two-photon calibration is inherently absolute, so there is no need for a lengthy calibration chain starting from a very high

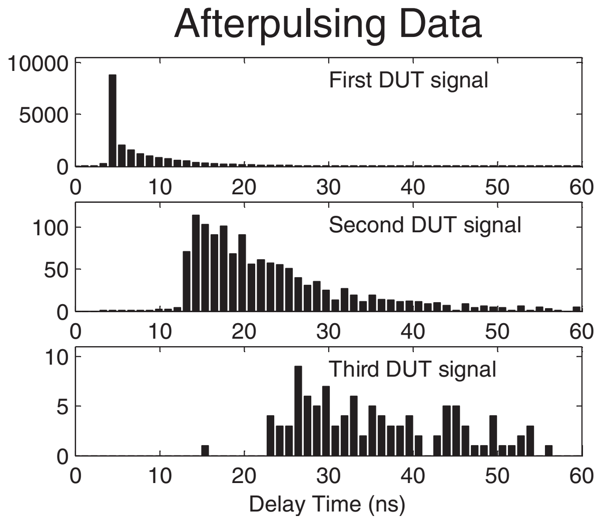

Figure 4. Histograms of the delay between the trigger and subsequent DUT signals. The top histogram records the delays of the first DUT signal event after the trigger, the middle records the second DUT signal event after the trigger, and the bottom histogram shows the third.

accuracy primary standard and extending through many steps (each with an associated degradation of uncertainty) to reach the final calibration. The coincidence technique naturally lends itself to calibrating photon-counting detectors, which are challenging to calibrate using conventional techniques (which work best for high flux analogue detectors). In PDC-based calibrations, the spectral selectivity does not need to be in the DUT optical path. This allows an additional advantage: one can more easily calibrate detectors in the infrared using visible bandpass filters.

These advantages demonstrate the value of the two-photon calibration technique. This technique has come a long way from the original feasibility studies, and shows considerable promise for continued improvements in the accuracy and reliability of the measurement method. It is clear that the technique offers considerable advantages in characterizing quantum cryptography systems and other related technologies. In such systems it can be important to know detection efficiencies and channel losses with a high degree of confidence, and the techniques of two-photon calibration naturally lend themselves to such measurements.

At NIST we are working with others to develop standard techniques for handling measurement issues such as those discussed in the previous section so that the method of two-photon calibration can be disseminated for general use, rather than just distributing physical reference standards for comparison (as is done in conventional calibration chains). As part of this effort we have developed a computer program to calculate PDC phase-matching parameters for various crystals that can be used to produce correlated photons (see http://physics.nist.gov/Divisions/Div844/facilities/cprad/cprad.html for details). We are anxious that this technique be moved out of the metrology lab and into the general community, where the end user can take maximum advantage of this inherently absolute measurement technique.

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