Achieving higher photon counting rates using multiplexed detectors

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ABSTRACT

As the quantum information field advances, the need for improved single-photon devices is becoming more critical. Quantum information systems are often limited by detector deadtime to count rates of a few MHz, at best. We present a multiplexed detection scheme that allows photon counting at higher rates than possible with single detectors. The system uses an array of detectors and an optical switch system to direct incoming photons to detectors known to be live. We model the system for realistic individual detector deadtimes and optical switching times. We show that such a system offers more promise than simply reducing the deadtime of an individual detector. We find that a system of N detectors with a given deadtime, can count photons at faster rates than a single detector with a deadtime reduced by $1/N$, even if it were practical to make such a large improvement.

Keywords: photon counting, detector, deadtime, quantum computing, quantum cryptography, single photon

1. INTRODUCTION

Quantum communication and quantum computation applications place difficult design requirements on the manipulation and processing of single photons.[1,2] Quantum cryptography[3] would particularly benefit from improved detectors, as applications such as Quantum Key Distribution (QKD) are often constrained by detector characteristics such as efficiency, dark count rate, timing jitter, and deadtime.[4] Because of demands for higher-rate secret key production, the quantum information community is presently engaged in a number of efforts aimed at improving QKD, including reducing detector deadtime.[5] Moreover, with the exponential growth in multimode parametric downconversion (PDC) photon pair production rates[6] now in the range of $2x10^6$ s^{-1} and the more recent development of $\chi^{(3)}$ single-mode fiber-based sources with pair rates[7,8] up to 10^7 s^{-1}, the need is clear for faster photon-counting detection. In typical single photon detectors presently available, either commercial or prototype, the deadtimes range from ≈50 ns for actively quenched avalanche photodiodes (APDs), to ≈10 μs for passively quenched APDs, although even actively quenched APDs sometimes employ μs deadtimes to avoid excessive afterpulsing rates. In addition to the absolute limits imposed by these effects, in practice detectors are often limited to small fractions of these rates (≈1 MHz) to avoid undesirable systematic effects associated with high deadtime fractions. Additional motivation for this proposal, is the improvement of traditional low light detection applications such as medical diagnosis, bioluminescence, and chemical and material analyses, where high speed and time resolution are also required.[9–12]

We present a scheme to improve detection rates by taking a pool of photon-counting detectors and operating them as a unit. The scheme consists of a 1-by-N optical switch that takes a single input stream of photons and distributes them to members of an array of N detectors. A switch controller monitors which detectors have fired recently and are thus dead, and then routes subsequent incoming pulses to a detector that is ready. We analyze and model and show that this system allows a system of N detectors to be operated at a significantly higher
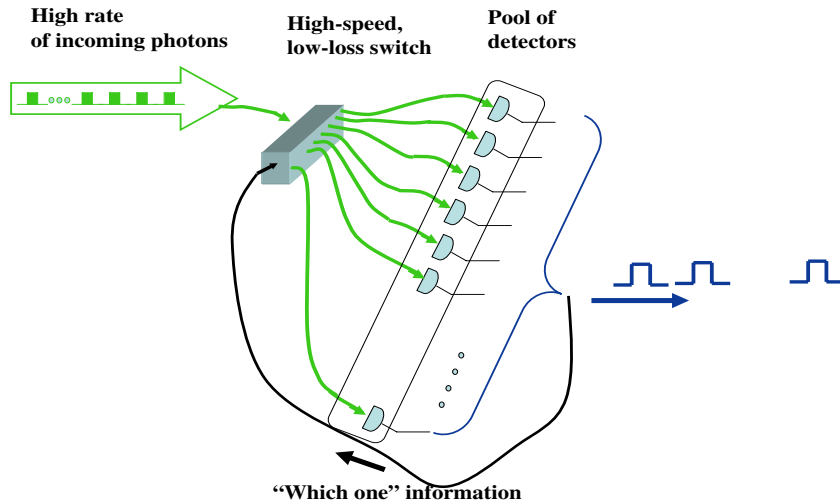
Figure 1. A pool of detectors and a fast switch are used to register a high rate of incoming photons. Incoming photons are switched to a ready detector. If it fires, the detector is switched out of the ready pool until recovery. If it does not fire, that detector remains ready.

detection rate than N times the detection rate of an individual detector, while maintaining the same Dead Time Fraction (DTF).

The system's switching operation could be sequential with each detector firing in order, or it could be set up to direct the input to any live detector. The later implementation may allow for optimum use of an array of detectors where each detector may have a different deadtime or when the switching time of the system is not negligible. In our model here we assume the detectors all have the same deadtime and switch transition time. (The switch transition time is meant to include any latency times, along with the actual optical transition times.) While we include switch latency time as part of the switch transition time rather than as a separate parameter, we point out that latency may affect the choice of what firing order minimizes transition times. For example, a system using a switch with a significant latency time (perhaps due to long processing times to determine if a detector has fired) might benefit from operation in a mode where, for a pulsed source, the input is switched to another live detector regardless of whether the previous detector fired. This would reduce the effect of the long latency as long as there is a high likelihood of there being at least one available live detector.

Compared to our previous analysis,[13] the main addition of our current effort is the inclusion of the effect of finite switching time. As before, we model the scheme where after a detector fires, the photons are sent by the optical switch to the next detector in the sequence of N-detectors. This is the simplest implementation and is all that is required when the optical switching time is not a large part of the overall system deadtime. For simplicity we retain this scheme here even as we allow the switching time to approach a significant fraction of the detector deadtime. In this new regime of non-negligible switch time, we assess the advantage of using this scheme versus passive schemes using passive beamsplitter trees with detector arrays,[14–16] as well as versus the performance of a single detector with much reduced deadtime. We show the superiority of the scheme over a hypothetical single detector with much improved deadtime, even when switching time is relevant. As previously, we define the DTF as the ratio of missed- to incident-events, and $R_{\mathrm{DTF}=10\%}$ as the rate of incoming photons giving a DTF=10%. This is often a reasonable limit for detector operation, so we use it as a benchmark to estimate of the performance of our scheme.

The theoretical model is done for an arbitrary number of detectors for a cw input source and a finite switching time of the control circuit. The goal here is to refine the theoretical model to include switch parameters that

could reasonably be expected for available devices. To achieve an analytical result we include the switching time as part of the overall system deadtime.

We tested the analytical modeling by comparison to Monte Carlo modeling designed to simulate an input stream of a large number of photons and numerically determine the DTF. Section 2 describes the theory for evaluating the DTF for cw poissonian distributed photons source for a system of N-detectors with a given switching time. The multiplexed detector system is considered as a single unit with an "effective" deadtime given by a statistically weighted contribution of the switching time plus the delay given by the single detector deadtime, when it is switched during its deadtime. The operation proceeds by switching the light input to the next detector after the previous detector registers a count. Section 3 describes the Monte Carlo simulation of the theory of section 2. Finally in section 4 we discuss the results and conclusions.

2. ANALYTICAL MODELING

Our analytical calculation estimates the DTF from the mean total count rate of the overall detector pool and effective deadtimes for each detector (which depend on their position in the switching system). We consider a Poissonian source and a pool of detectors with the identical detection efficiencies η and identical non-extending deadtimes T_{d}.[17] (We refer to a Poissonian source as cw because, while the photons arrive at discrete times, they have equal probability to arrive at any time.)

The probability that n photons from a Poissonian source with mean photon rate λ are registered by a single live detector with efficiency η in a time interval T is $P(n) = (\eta\lambda T)^n e^{-\eta\lambda T}/n!$. Thus the mean number of counts registered is $\eta\lambda T$. From here on for simplicity we assume $\eta = 1$. (We can do this without loss of generality, as η and λ always appear together and can thus be traded off against each other without affecting the ultimate results.) In the presence of deadtime T_{d} and for measurement time $T \gg T_{\mathrm{d}}$, the mean number of counts registered reduces to

$$M = \lambda T - M\lambda T_{\mathrm{d}}. \tag{1}$$

Rearranging, we have

$$M = \frac{\lambda T}{1 + \lambda T_{\mathrm{d}}}. \tag{2}$$

The DTF, defined as the ratio of the lost counts over the total counts in the absence of dead time, for this simple case is

$$\mathrm{DTF} = \frac{\lambda T - M}{\lambda T} = 1 - \frac{1}{1 + \lambda T_{\mathrm{d}}}. \tag{3}$$

Here we consider the array of N detectors as a unified detection resource, having an overall or "effective" deadtime $\mathcal{T}_{\mathrm{d}(N)}$. Therefore the DTF is

$$\mathrm{DTF} = 1 - \frac{1}{1 + \lambda\mathcal{T}_{\mathrm{d}(N)}}. \tag{4}$$

To highlight its advantage we will compare this N-detector system DTF to that which could be achieved by of a single detector with a deadtime reduced by a factor of $1/N$. For such an improved single detector, $\mathrm{DTF} = 1 - \frac{1}{1 + \lambda T_{\mathrm{d}}/N}$. This is also the same result that would be obtained by an array of N detectors with deadtime T_{d} and passive switching such as may be implemented with a tree arrangement of beam splitters.

We now calculate the effective deadtime of the system. Because the optical switch only switches photons to a new detector after a registered count, the effective deadtime can be given by the statistical contribution of the switching time, T_{s} and the single detector dead time, T_{d}, governed by two cases- either a) N events are counted in a time interval bigger than $T_{\mathrm{d}} - T_{\mathrm{s}}$, or b) they occur in a time interval less than $T_{\mathrm{d}} - T_{\mathrm{s}}$. In the second case, the photon is switched to a dead detector adding an additional delay to the optical switching time. We write the effective deadtime for N detectors as

$$\mathcal{T}_{\mathrm{d}(N)} = p_{a,N}(\mathcal{T}_{\mathrm{d}(N)})T_{\mathrm{s}} + p_{b,N}(\mathcal{T}_{\mathrm{d}(N)})(T_{\mathrm{d}} - E_{b,N}(\mathcal{T}_{\mathrm{d}(N)})), \tag{5}$$

where

$$p_{a,N}(\mathcal{T}_{\mathrm{d}(N)}) = \int_{T_{\mathrm{d}}-T_{\mathrm{s}}}^{+\infty} f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)}) \mathrm{d}\Delta t, \tag{6}$$

and

$$p_{b,N}(\mathcal{T}_{\mathrm{d}(N)}) = \int_0^{T_{\mathrm{d}}-T_{\mathrm{s}}} f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)}) \mathrm{d}\Delta t, \tag{7}$$

are the probabilities that case (a) or (b) occurs for $f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)})$, the probability density distribution of the time interval Δt, between a count and the $N-1^{th}$ preceding one. We indicate the dependence of the above probabilities on $\mathcal{T}_{\mathrm{d}(N)}$.

$$E_{b,N}(\mathcal{T}_{\mathrm{d}(N)}) = \frac{\int_0^{T_{\mathrm{d}}-T_{\mathrm{s}}} \Delta t f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)}) \mathrm{d}\Delta t}{\int_0^{T_{\mathrm{d}}-T_{\mathrm{s}}} f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)}) \mathrm{d}\Delta t} \tag{8}$$

is the mean time interval between a count and the $N-1^{th}$ preceding one when case (b) occurs. For a poissonian process where events are counted with an overall deadtime of fixed length $\mathcal{T}_{\mathrm{d}(N)}$, $f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)})$ is given by[17]

$$f_N(\Delta t, \mathcal{T}_{\mathrm{d}(N)}) = \frac{\lambda^{N-1}[\Delta t - (N-1)\mathcal{T}_{\mathrm{d}(N)}]^{N-2}}{(N-2)!} \exp(-\lambda[\Delta t - (N-1)\mathcal{T}_{\mathrm{d}(N)}])\theta[\Delta t - (N-1)\mathcal{T}_{\mathrm{d}(N)}], \tag{9}$$

which is a modified Gamma function, and θ is the Heaviside step function with $\theta(x) = 1$ for $x > 0$ and 0 otherwise. Only in the case of $N = 2$ is the effective deadtime explicitly calculable. For more detectors we use numerical methods.

In our previous work[13] where the switching time was neglected, an explicit formula was possible. As it should, the current model coincides with our previous calculation in the limit of switching time short compared to the single detector deadtime. This theoretical approach is also in agreement, as we will show, with Monte-Carlo simulation results.

2.1. Monte Carlo Modeling

The Monte Carlo modeling of our detection scheme for a cw source uses Poisson distributed incident photons at a range of rates meant to describe the use of the system in conjunction with a laser source. As mentioned before, we can assume 100 % efficient collection and detection without loss of generality. The individual detector deadtimes were set to 1 μs. The modeling procedure consisted of first using a random number generator to simulate an input stream of a large number of photons with exponentially distributed arrival times, which corresponds to a poissonian photon number distribution with a given mean photon number. The resulting photon arrival list was then apportioned to the detectors according to our switching plan. (We note that because of the inclusion of finite switching times, we cannot use the same iterative Monte Carlo procedure that was employed in our previous paper[13] for the zero switching time situation.) In the case here with finite switching time, we go through the time-sorted list of photon arrival times sending the first photon to the first detector, we skip any photons within one switching time after that count, then proceed to the next incoming count which is recorded by the next detector, and repeat these steps while also keeping track of if the next detector has recovered from its most recent firing. On completing this process for the entire photon arrival list we have sorted the photons into those detected and those missed to get an overall DTF.

3. RESULTS

Figure 2 (a) shows the dead time fraction for $N = 1$ to 5 detectors with switching times of 1% and 10% of the single detector deadtime, versus the incoming photon rate, for a single detector deadtime of 1 μs. For $T_{\mathrm{s}} = 0.1T_{\mathrm{d}}$, the multiplexed scheme shows much less increase of the $R_{\mathrm{DTF}=10\%}$ points with increasing detector number. For $T_{\mathrm{s}} = 0.01T_{\mathrm{d}}$ the effect of switching time on the system is negligible. Fig. 2 (b) compares the analytic theory with the Monte Carlo results for one case of fig. 2 (a). The theory and simulation agree well for all switching times considered. Fig. 2 (c) compares the active multiplexed scheme of this paper with a passive scheme (detector/beamsplitter tree configuration) for $T_{\mathrm{s}} = 0.1T_{\mathrm{d}}$. As judged by the $R_{\mathrm{DTF}=10\%}$ points, the active multiplexed scheme surpasses the passive arrangement for $N = 4, 5$.
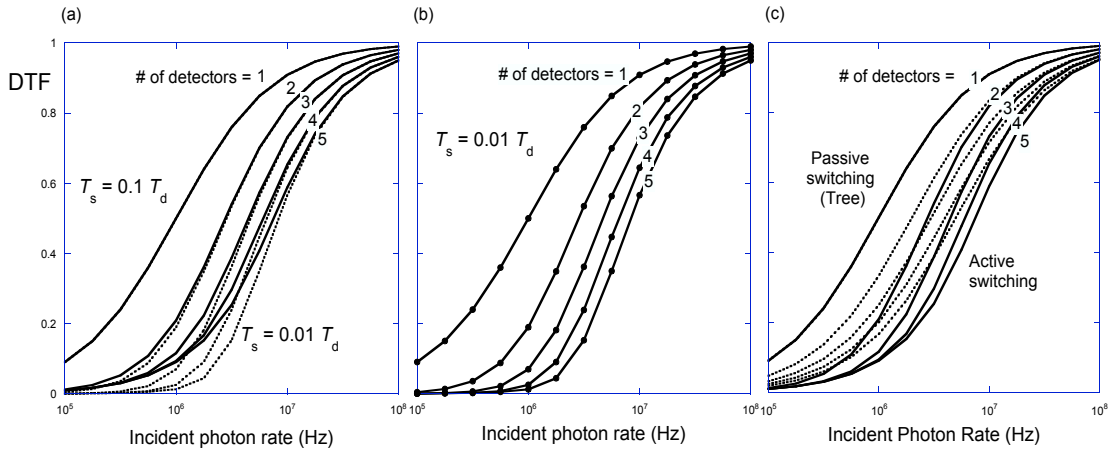
Figure 2. DTF versus the incoming photon rate for $N=1$ to 5 detectors with $T_d=1$ μs (a) for $T_s = 0.01$ T_d (dotted lines) and $T_s = 0.1$ T_d (solid lines). (b) for theoretical (solid lines) and Monte Carlo (points) results for $T_s = 0.01$ T_d. (c) for $T_s = 0.1$ T_d for a passive scheme (dotted lines) and active switched scheme (solid lines).
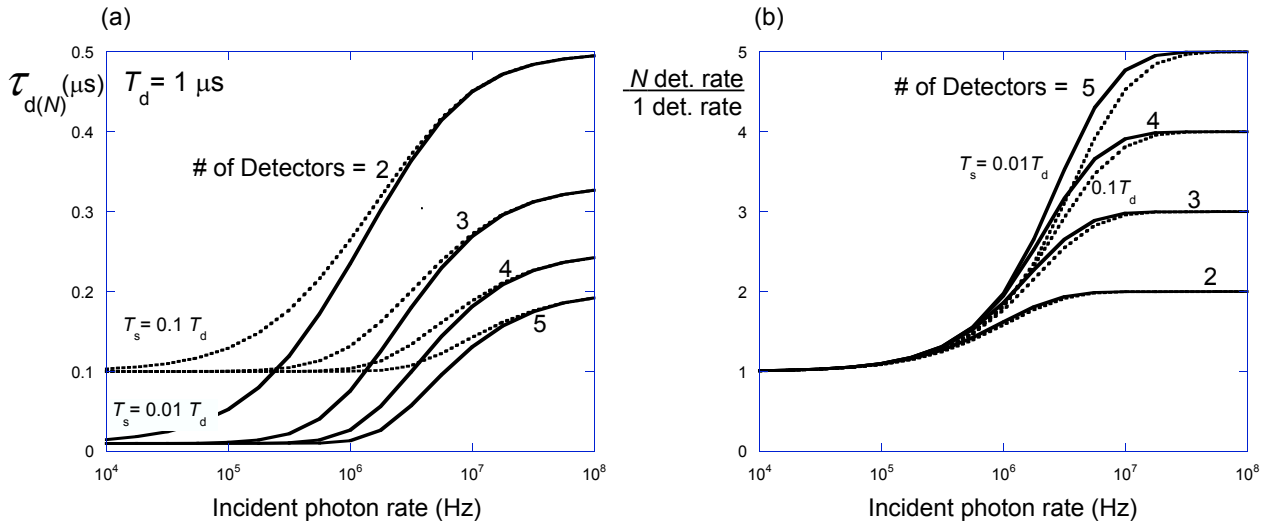

Figure 3. For systems of 2 to 5 detectors all with $T_d = 1$ μs (a) Plots of $\mathcal{T}_{d(N)}$ with $T_s = 0.1$ T_d (dotted lines) and $T_s = 0.01$ T_d (solid lines). The limit for $\mathcal{T}_{d(N)}$ for high incoming photon rate is T_d/N, while for low photon rate it is T_s. (b) Ratio of the number of photons counted by multiplexed systems to those counted by a single detector, all for $T_d= 1$ μs, and $T_s = 0.1$ T_d (dotted lines), $T_s = 0.01$ T_d (solid lines). The ratio limit for high incoming photon rate is N.

Figure 3 (a) shows the mean effective deadtime $\mathcal{T}_{d(N)}$ for N up to 5, versus the mean incident photon rate (λ), for $T_s = 0.1$ T_d and 0.01 T_d. The effective deadtime clearly satisfies the condition $T_s \leq \mathcal{T}_{d(N)} \leq T_d/N$. We see that the maximum effective deadtime of the multiplexed scheme coincides with the detector tree deadtime. This means that for an optical switch with $T_s < T_d/N$, our scheme surpasses what is possible with a passive scheme.

Figure 3 (b) shows the ratio of the mean count rate for our multiplexed scheme to the count rate of a single detector, versus the mean incoming photon rate. We see that as expected for high count rate the maximum gain is N-times the rate that would be obtained by a single detector.

Figure 4 shows $R_{\mathrm{DTF}=10\%}$ versus the number of detectors for the active switching system for several switching
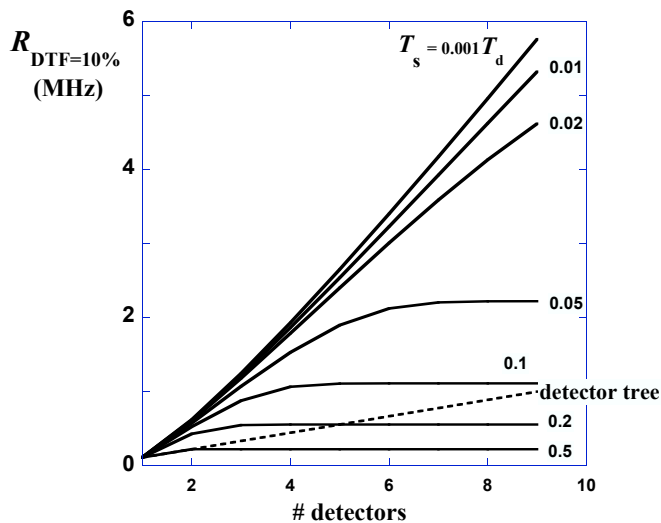
Figure 4. Incoming rate giving a DTF=10% ($R_{\mathrm{DTF=10\%}}$) versus the number of detectors, for $T_\mathrm{s} = 0.001$, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5 of T_d (solid lines). The detector tree configuration result is also shown (dashed line).

times. The $T_\mathrm{s} = 0.001\ T_\mathrm{d}$ result differs little from the theory that neglected the switching time.[13] Up to $T_\mathrm{s} = 0.02\ T_\mathrm{d}$ the results show significant advantage of the active switch scheme for all numbers of detectors shown. Above $T_\mathrm{s} = 0.2\ T_\mathrm{d}$ the advantage is reduced until ultimately the active system falls below the passive scheme for just a few detectors.

4. CONCLUSION

We have shown that a pool of N detectors with a controlled switch system can in principle be operated at much higher incident photon rates than is otherwise possible either with a single detector with much reduced deadtime, or an array of detectors with a passive switch system such as might be implemented with by a tree of beamsplitters. Our modeling included realistic optical switch transition times and showed that switch transitions times are negligible when they are less than 2% of the individual detector deadtimes which means that the scheme should be practical for the case of detectors like InGaAs APDs which often operate with microsecond deadtimes. However for the switching times of 20% or more of the single photon detector deadtime, a detector tree configuration would be more convenient and advantageous.

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