

Studying Photon Number Distribution of NV-Centre Emission in Nano-Diamond

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Motivation

Application of single photon sources ...

- Quantum physics
- Quantum computing
- Quantum Key Distribution
- Biomedical physics
- Efficiency calibration of single photon detectors

Motivation

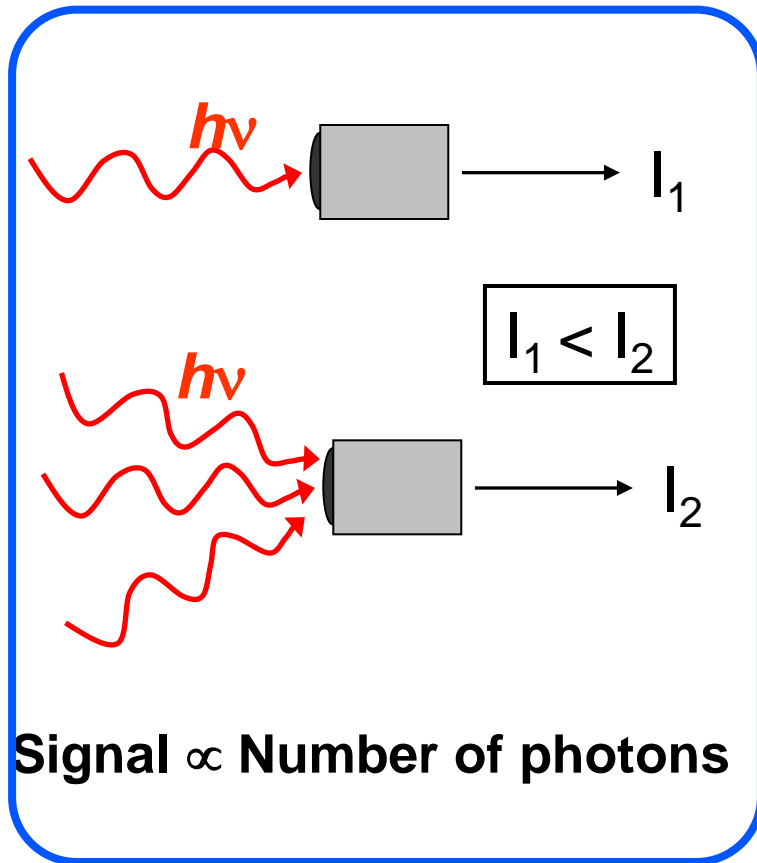
Why photon number distribution is of interest?

e.g.

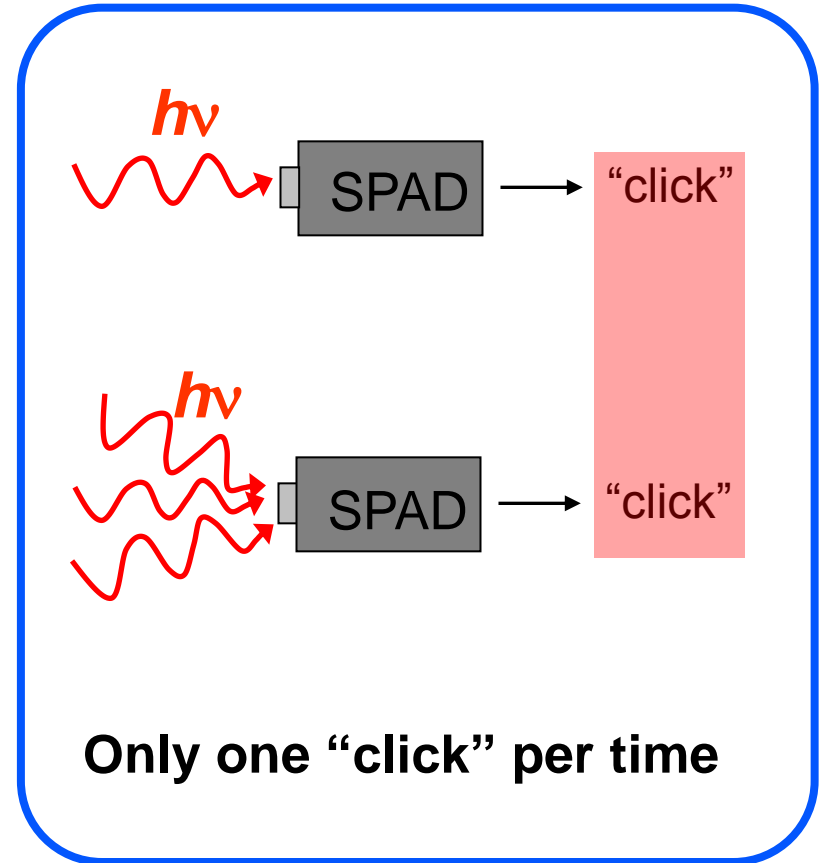
- quantum key distribution:
 - multiphoton events vulnerable to attacks
- efficiency calibration of single photon detectors
 - photon statistics affects result of calibration

Single Photon Detector Calibration

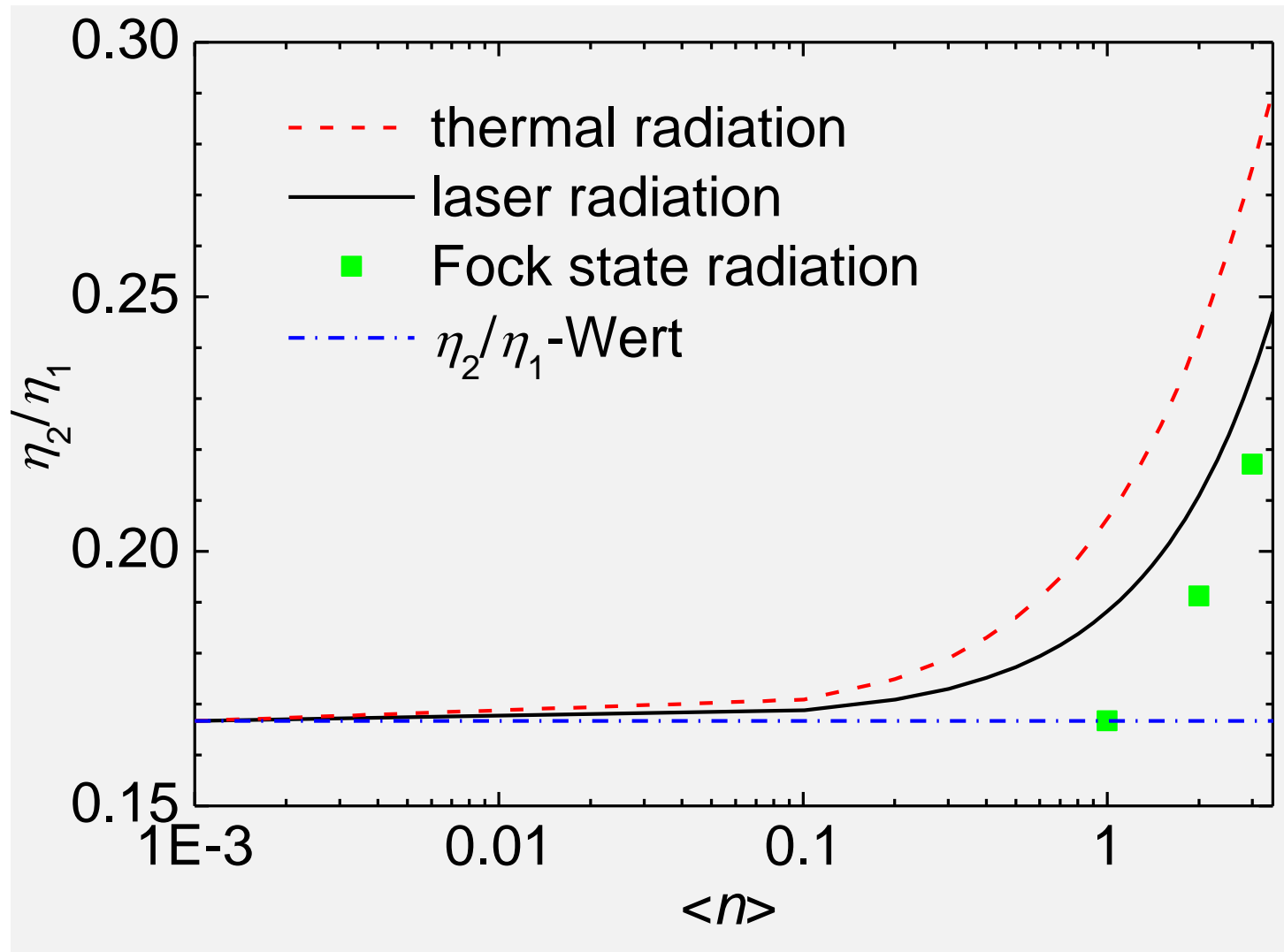
Analogue detector



Digital detector



Single Photon Detector Calibration



Introduction

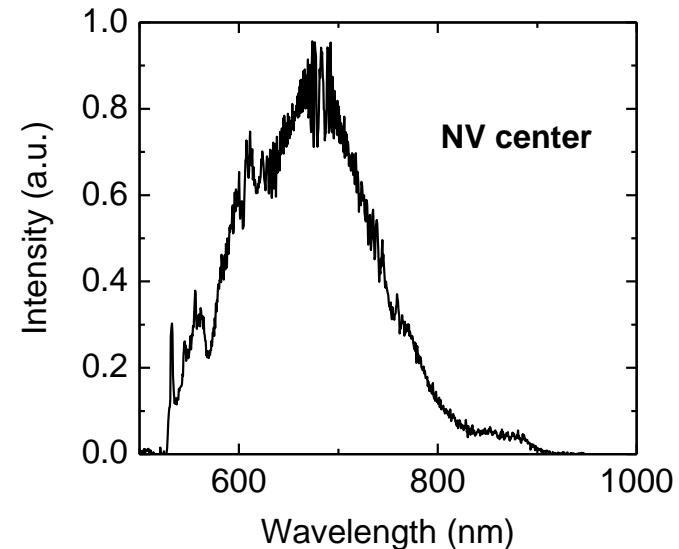
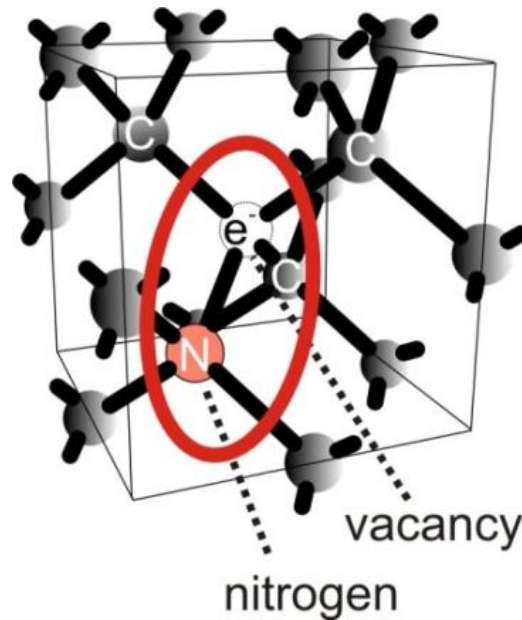
Several methods have been developed to analyze unknown photon statistics, e.g.:

- photon number resolving detector, e.g. transition edge sensor (TES)
- “on/off-measurements”
- detector tree
- ...

Here:

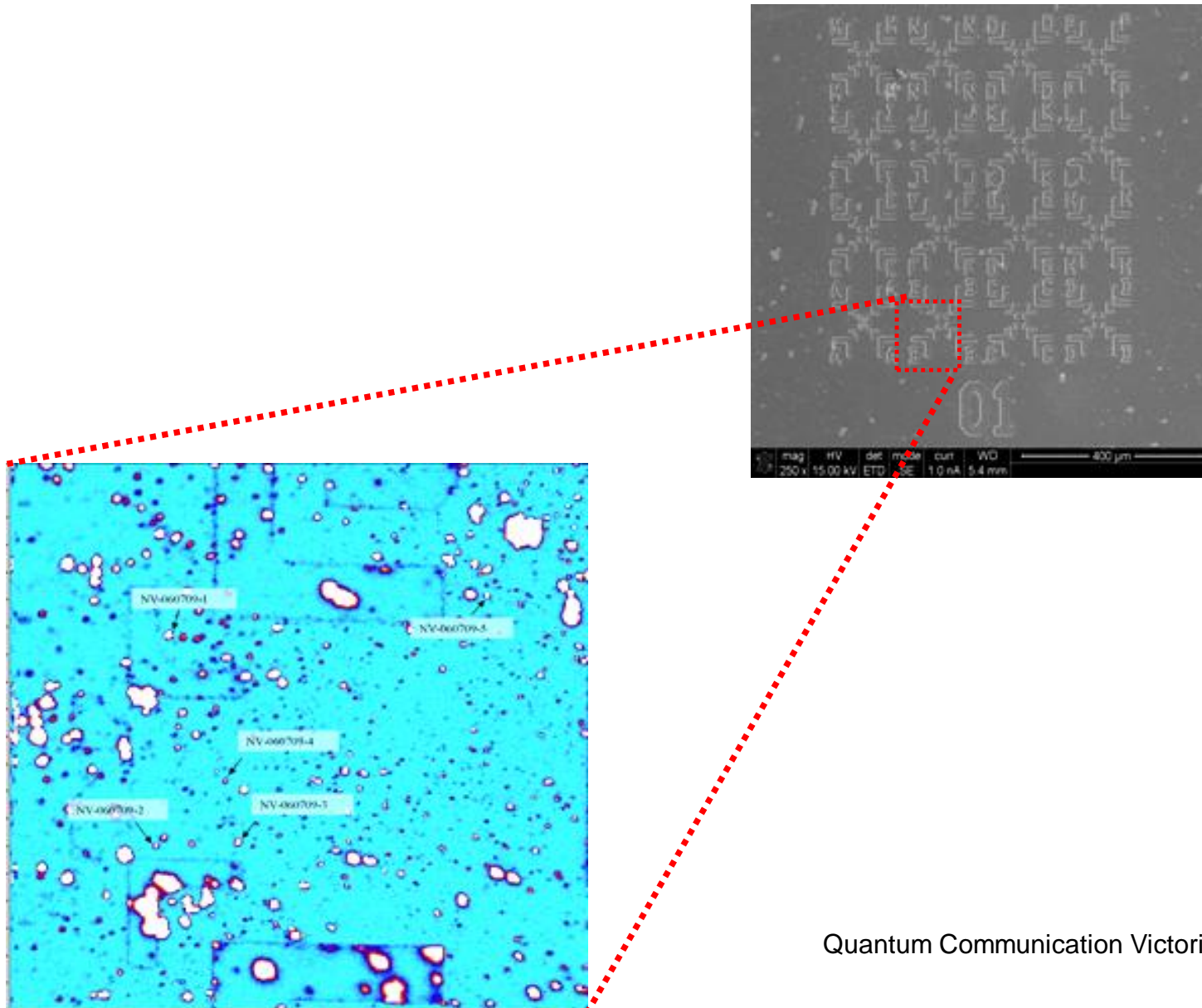
- NV-centre emission is studied
- results compared to the $g^{(2)}(0)$ -values obtained by Hanbury-Brown & Twiss (HBT)-measurements

The Nitrogen-Defect (N/V) Center in Diamond

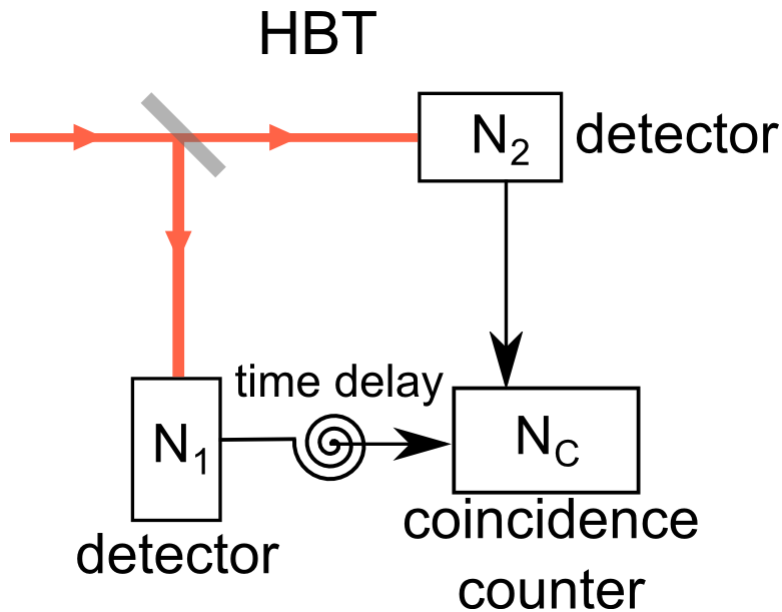


- high photo stability at room temperature
- bright emission: up to 10^6 photons per second detected
- short decay time 8 ns - 30 ns
- broad luminescence spectrum
- NV-defects in **nano diamonds**: minimization of optical limitations caused by the high refractive index of diamond

Nano-diamonds with NV-centres on Si substrate



Hanbury-Brown Twiss Interferometer



$$g^2(\tau) = \frac{N_c}{N_1 \cdot N_2 \cdot \Delta\tau \cdot T}$$

Second order correlation function

$$g^2(t) = \frac{\langle I(t+\tau) I(t) \rangle}{\langle I(t) \rangle^2}$$

$$g^{(2)}(0) = 1 - \frac{1}{n}$$

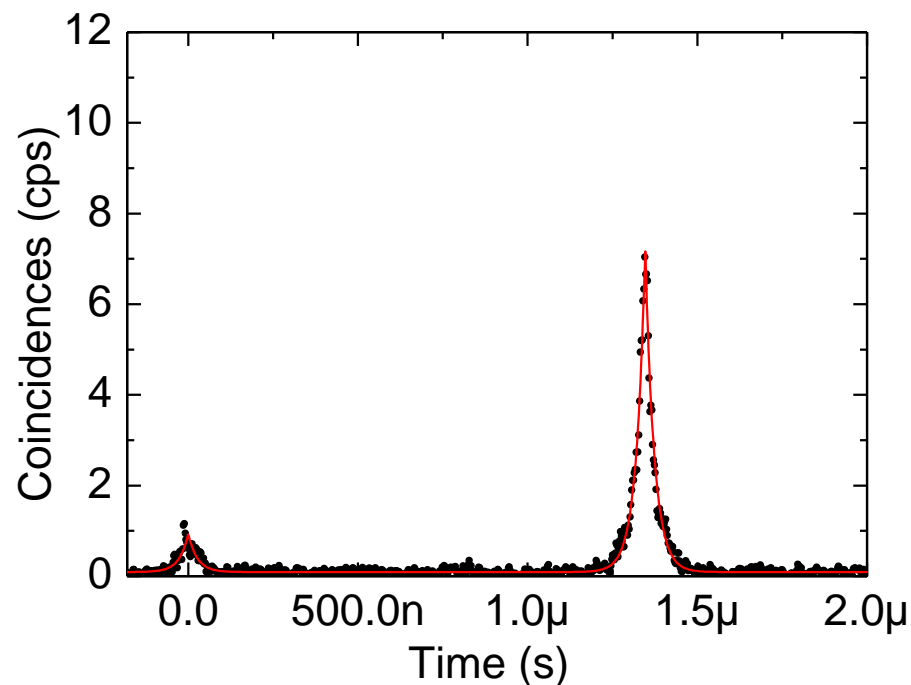
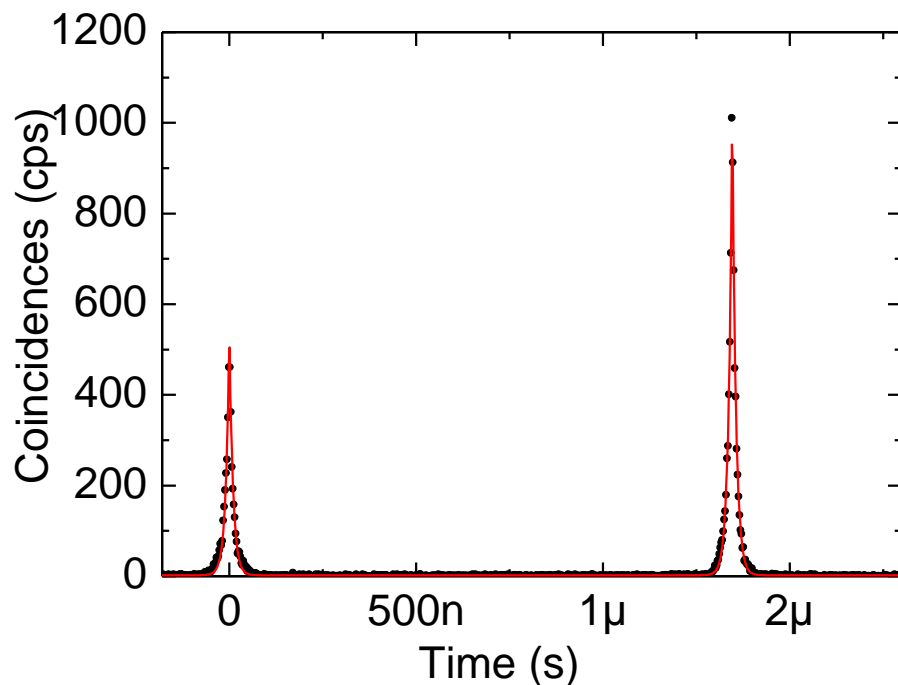
Number of single photon emitters

HBT Interferometer Measurements

Hanbury-Brown-Twiss set up $\rightarrow g^{(2)}(t)$

Centre 1: $g^{(2)}(0) = 0.53 \pm 0.02$

Centre 2: $g^{(2)}(0) = 0.06 \pm 0.03$



excitation: pulsed laser @532 nm, 743 kHz
emission: spectral filtering: (694 ± 37) nm

Photon Number Resolving Detector at INRIM

Transition edge sensor (TES):

Sensitive calorimeter measuring pulse energy

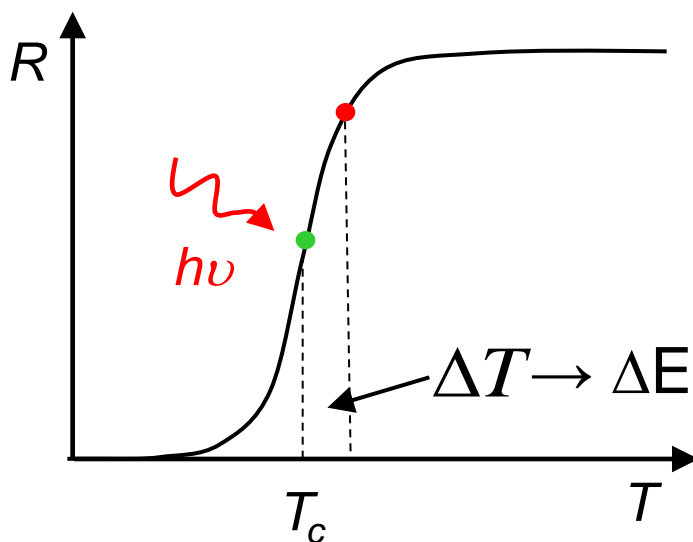
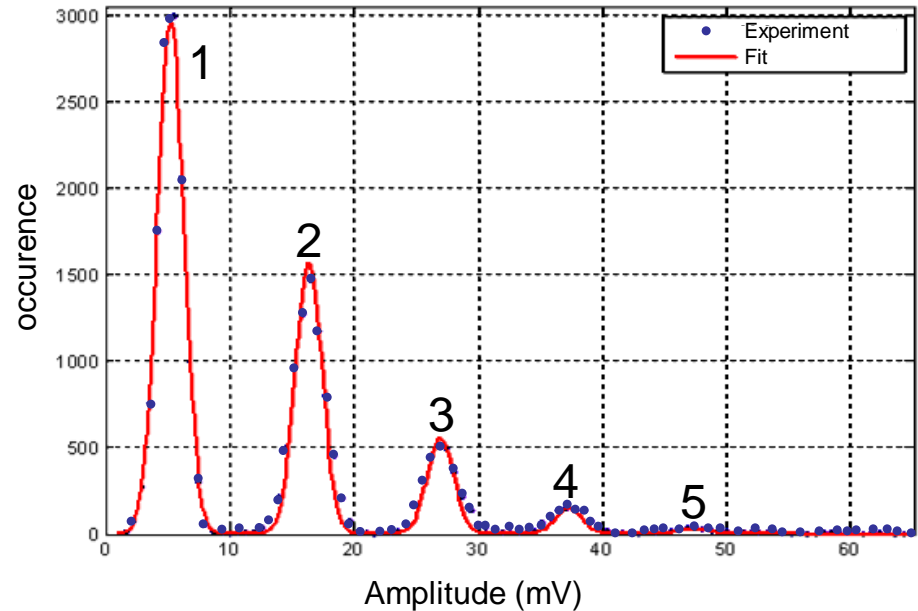
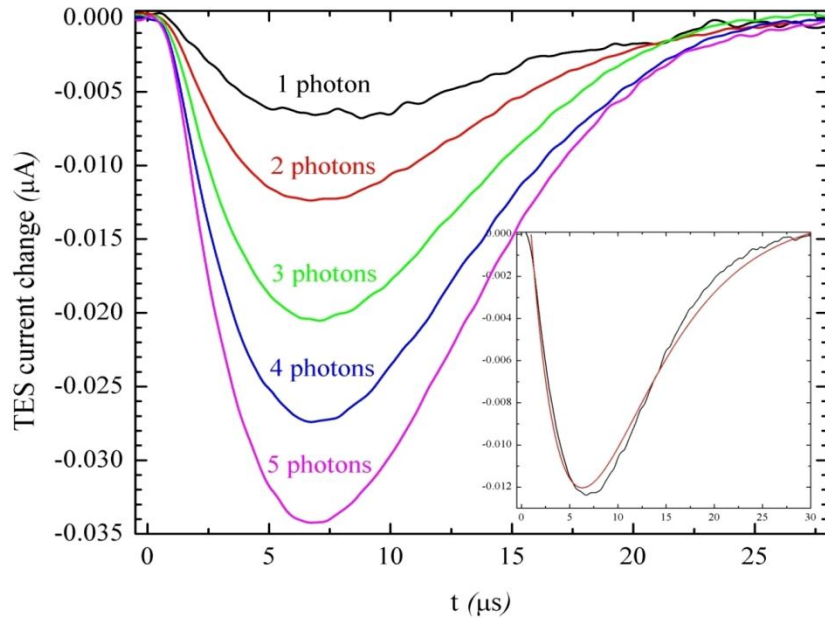


Foto: G.Brida

- Operation temperature $T_c = 130$ mK
- Sharp superconducting phase transition $\Delta T \sim h\nu = 1$ mK
- Optical fibre coupling (9 μm core)



Pulse amplitude $\sim n/\text{wavelength}$

Time constants

- electrical rise time $\sim 5.2 \mu\text{s}$
- thermal response time $\sim 32 \mu\text{s}$

Energy resolution

$$\Delta E = 0.41 \text{ eV @ } 690 \text{ nm}$$

TES – Measurement Results

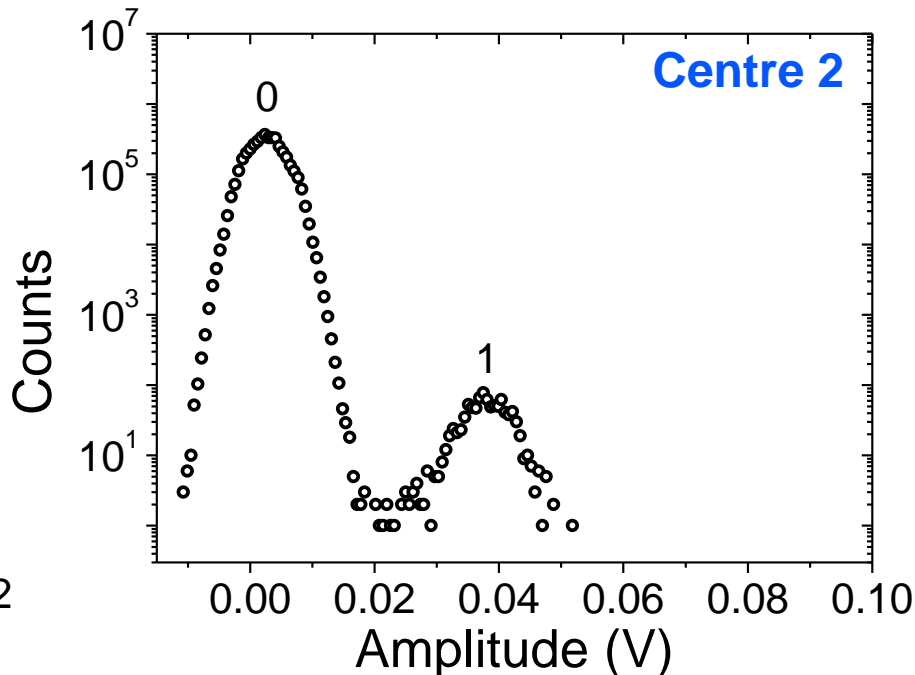
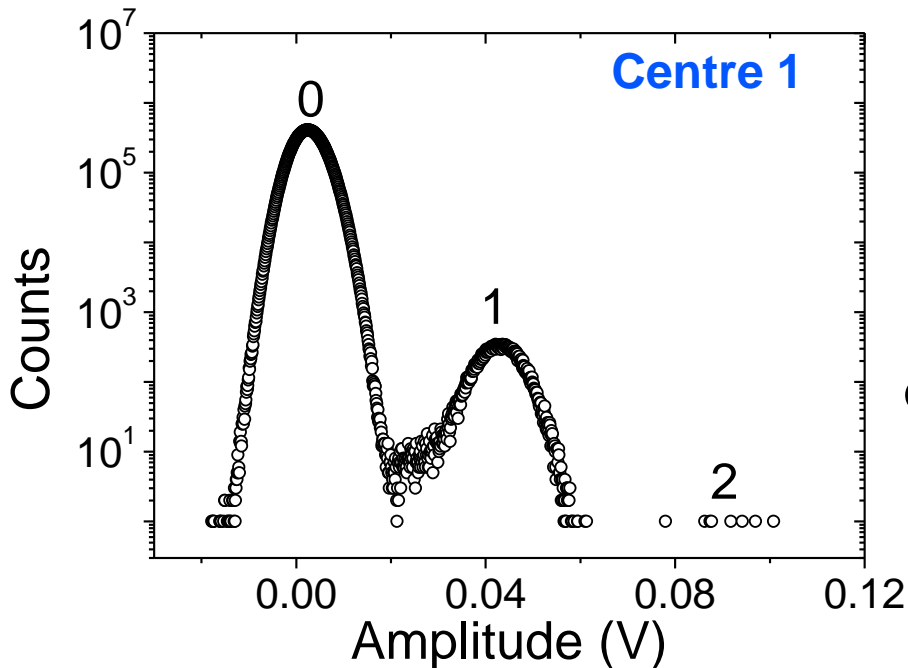
$g^{(2)}(0)$ -values: TES results agree with values measured by the HBT

TES: $g^{(2)}(0) = 0.5 \pm 0.1$

HBT: $g^{(2)}(0) = 0.53 \pm 0.02$

TES: $g^{(2)}(0) = 0.0 \pm 0.1$

HBT: $g^{(2)}(0) = 0.06 \pm 0.03$



$$g^{(2)}(0) = \frac{\sum_n (n^2 P_n - n P_n)}{(\sum_n n P_n)^2}$$

'On/Off' Photon Statistic Reconstruction

Partial reconstruction of photon statistics by varying detection efficiency

'No-click' probability p_0 for a binary detector of detection efficiency η_v hit by a quantum optical field with density matrix ρ :

$$p_0(\eta_v) = \sum_n (1 - \eta_v)^n \underbrace{\langle n | \rho | n \rangle}_{\text{diagonal elements of the density matrix}}$$

diagonal elements
of the density matrix

Photon number distribution and $g^{(2)}(0)$ -value can be obtained by:

- Maximum Likelihood estimation
- least square fit

'On/Off' Measurements

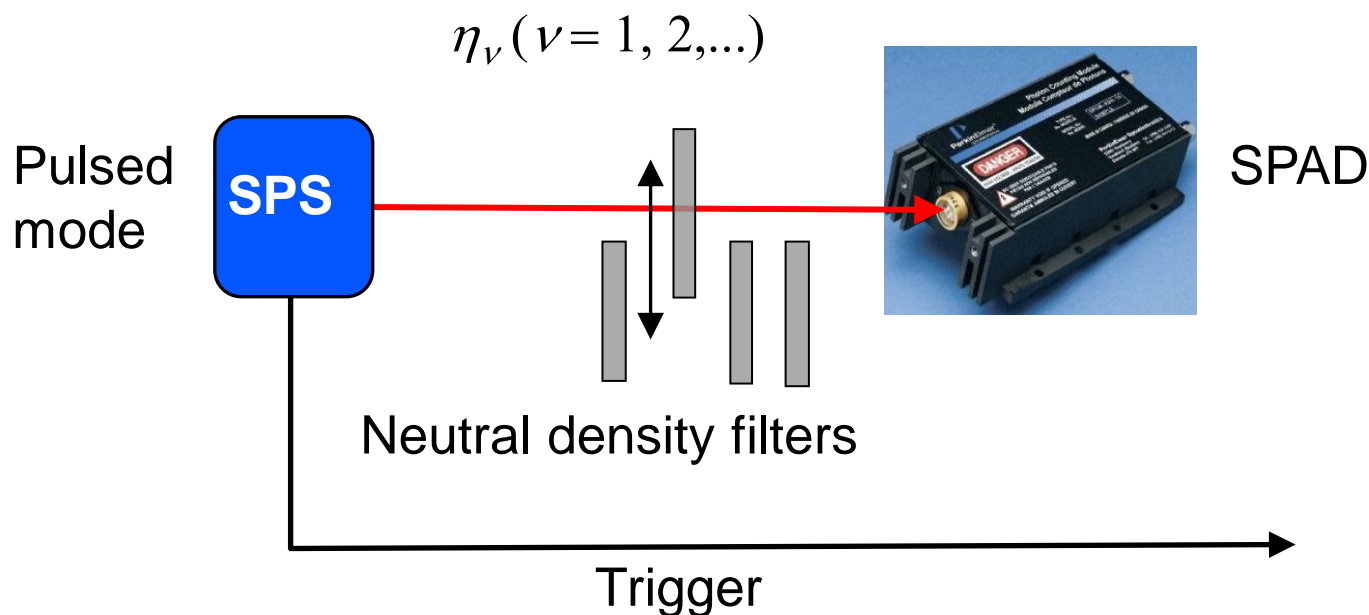
Information on the photon number distribution is gained from the ratio:

$$f_v(\eta_v) = \frac{n_{0,v}}{n_v}$$

variable detection efficiency

$n_{0,v}$: number of "no-click" events

n_v : total number of runs



'On/Off' Measurements Results

"On/Off": $g^{(2)}(0) = 0.5 \pm 0.1$

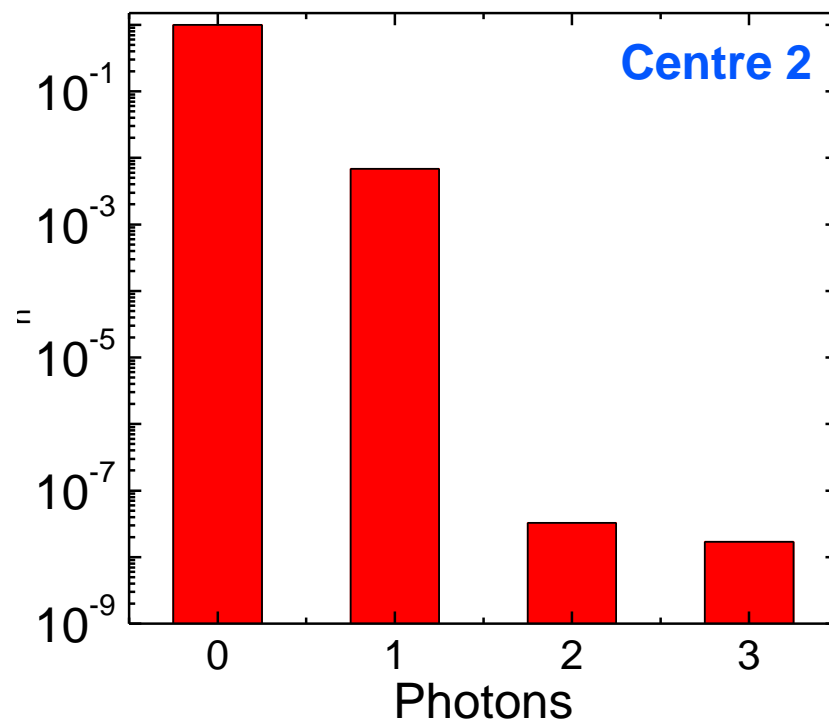
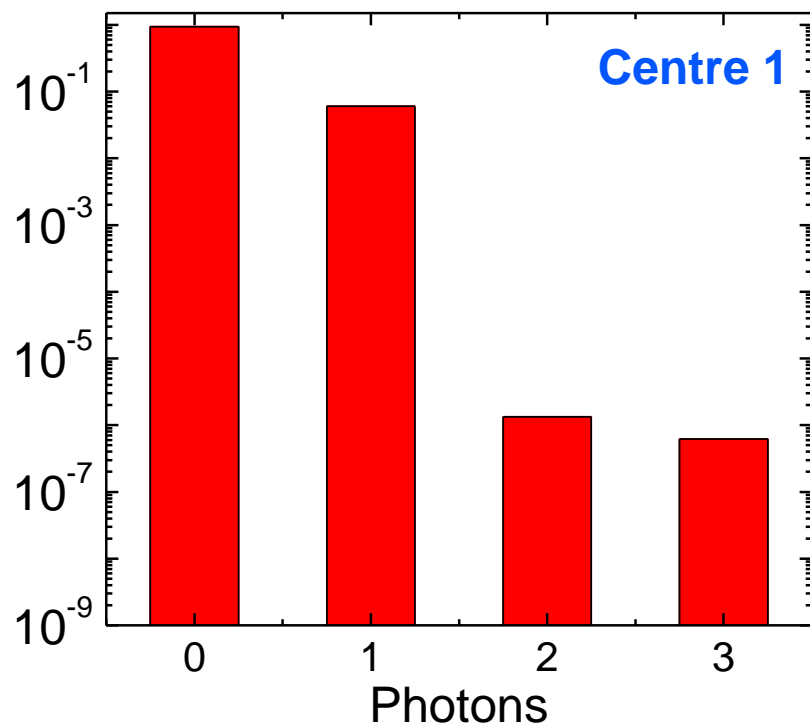
TES: $g^{(2)}(0) = 0.5 \pm 0.1$

HBT: $g^{(2)}(0) = 0.53 \pm 0.02$

"On/Off": $g^{(2)}(0) = 0.02 \pm 0.05$

TES: $g^{(2)}(0) = 0.0 \pm 0.1$

HBT: $g^{(2)}(0) = 0.06 \pm 0.03$



$$g^{(2)}(0) = \frac{\sum_n (n^2 P_n - n P_n)}{(\sum_n n P_n)^2}$$

Photon number distribution of NV-centre emission ...

- measured by TES detector and ‘On/Off’ measurements
- results exhibit high zero photon components
 - low overall efficiency of the used single photon source
- $g^{(2)}(0)$ -value calculated from photon number distribution compared to HBT interferometer values
 - good agreement for TES- and “On/Off”-results

Further ‘On/Off’ measurements will be performed for brighter NV-centres

→ to study the applicability of the method for determination of the photon statistics of single photon sources in general

Single Photon Detector Calibration

$$Q_{\text{Laser}}(\langle \bar{n} \rangle) = \sqrt{\frac{(1 - e^{-\alpha\eta_D \langle \bar{n} \rangle})(1 - e^{-(1-\alpha)\eta_D \langle \bar{n} \rangle})}{(1 - e^{-\alpha\eta_S \langle \bar{n} \rangle})(1 - e^{-(1-\alpha)\eta_S \langle \bar{n} \rangle})}}$$

$$Q_{\text{therm.}}(\langle \bar{n} \rangle) = \frac{\eta_D}{\eta_S} \sqrt{\frac{(1 + \alpha\eta_S \langle \bar{n} \rangle)(1 + (1 - \alpha)\eta_S \langle \bar{n} \rangle)}{(1 + \alpha\eta_D \langle \bar{n} \rangle)(1 + (1 - \alpha)\eta_D \langle \bar{n} \rangle)}}$$

$$Q_{\text{Fock}} \langle \bar{n} \rangle = \sqrt{\frac{(1 - (1 - \alpha\eta_D)^{\langle \bar{n} \rangle})(1 - (1 - (1 - \alpha)\eta_D)^{\langle \bar{n} \rangle})}{(1 - (1 - \alpha\eta_S)^{\langle \bar{n} \rangle})(1 - (1 - (1 - \alpha)\eta_S)^{\langle \bar{n} \rangle})}}$$

$$Q_{\text{SPS}} = \frac{\eta_D}{\eta_S}$$

'On/Off' Measurements

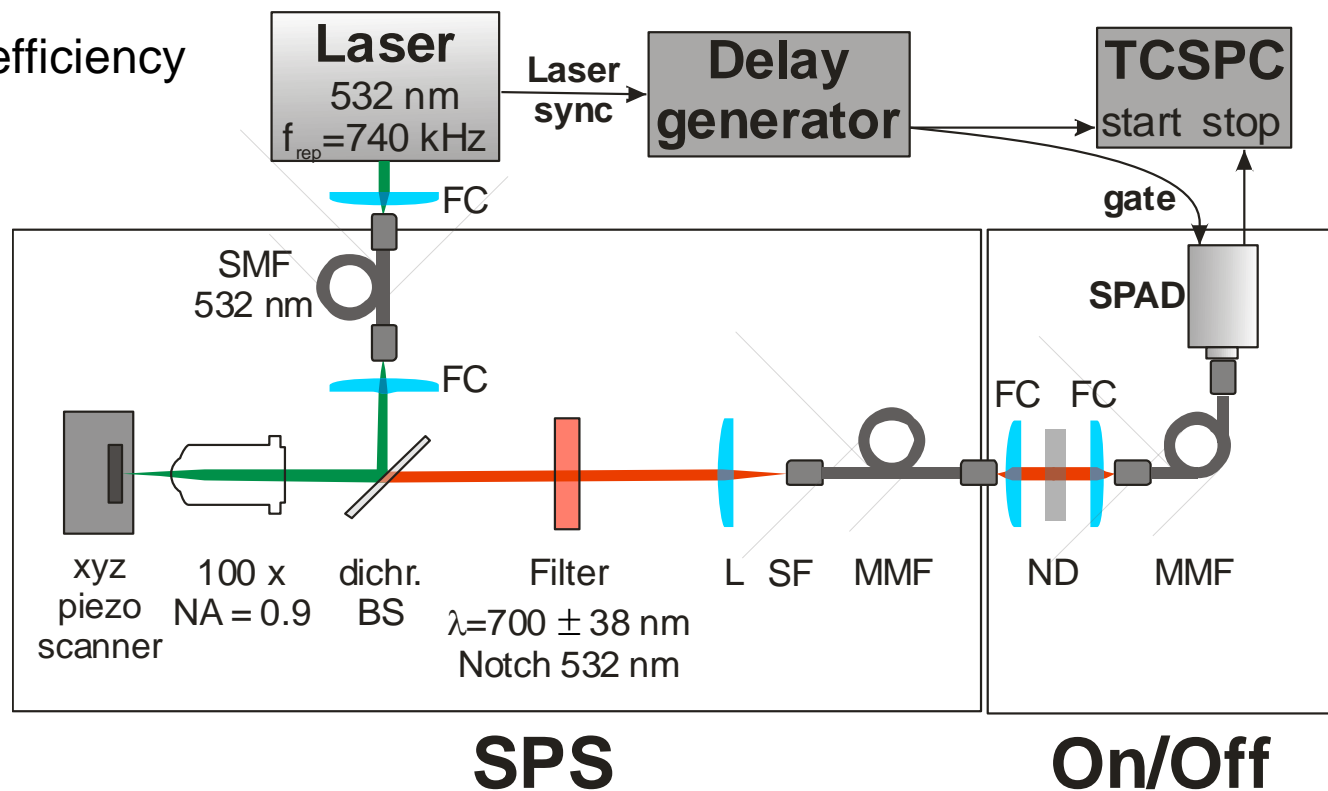
Information on the photon number distribution is gained from the ratio

$$f_v(\eta_v) = \frac{n_{0,v}}{n_v}$$

$n_{0,v}$: number of "no-click" events

n_v : total number of runs

variable detection efficiency



Photon Statistics Reconstruction by On/Off measurements

Non-click probability for an on/off detector (APDs, ecc.) of quantum efficiency η_v hit by a quantum optical field with density matrix ρ :

$$P_v = p_0(\eta_v) = \sum_n A_{v n} \rho_n \quad \left\{ \begin{array}{l} A_{v n} = (1 - \eta_v)^n \quad \eta_v = \text{quantum efficiency} \\ \rho_n = \langle n | \rho | n \rangle \quad \rho = \text{density matrix} \end{array} \right.$$

Maximum-likelihood based **constrained Log-likelihood probability function** (on a truncated N -dimensional Hilbert space for the quantum optical field):

$$L_\beta = \sum_{v=1}^K f_v \cdot \log \left(\frac{P_v}{\sum_\lambda P_\lambda} \right) - \sum_{n=1}^N \beta n \rho_n \quad \left\{ \begin{array}{l} f_v = \text{fraction of non-click events} \\ \beta = \text{energy constraint} \end{array} \right.$$

Obtained iterative solution (the apex (i) indicates something depending on the ρ_n at the i -th iteration):

$$\frac{\partial L_\beta}{\partial \rho_n} = 0 \quad \Rightarrow \quad \rho_n^{(i+1)} = \frac{\rho_n^{(i)}}{\sum_m \rho_m^{(i)}} \cdot \sum_{v=1}^K \left(\frac{A_{v n}}{\sum_\lambda A_{\lambda n} + \beta n \left(\frac{\sum_r P_r^{(i)}}{\sum_r f_r} \right)} \cdot \frac{f_v}{P_v^{(i)}} \right)$$

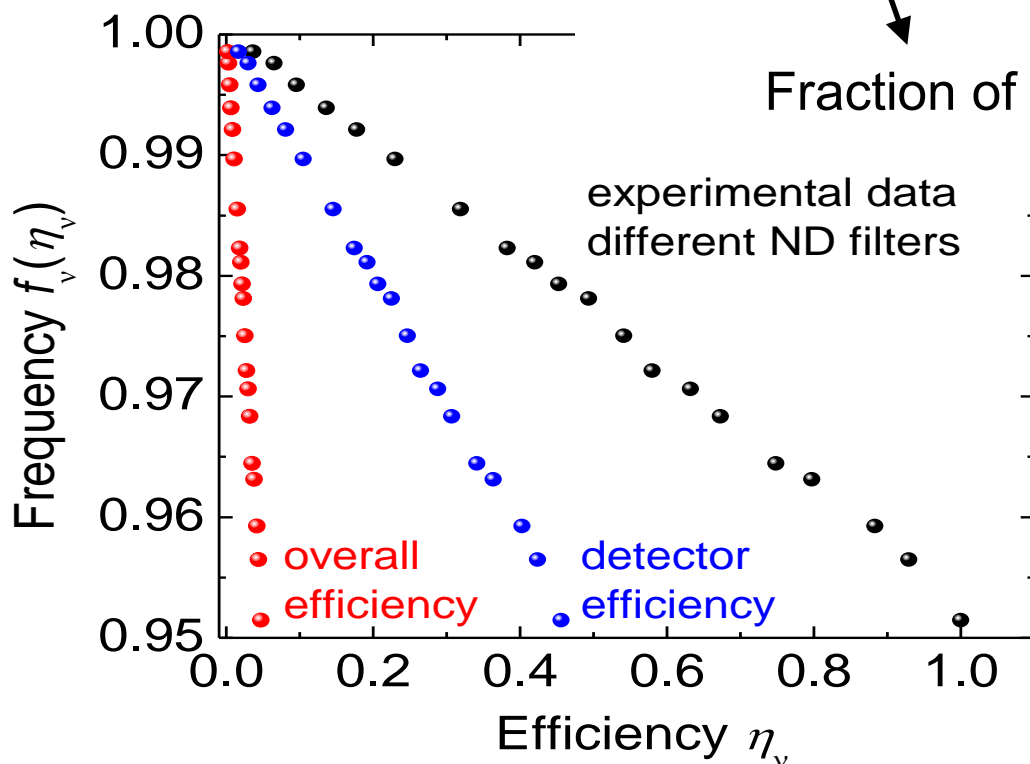
Maximum-likelihood estimation

... based on constrained **Log-likelihood probability** function

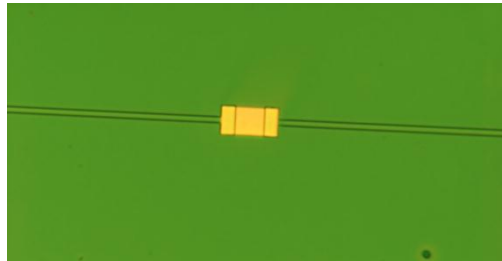
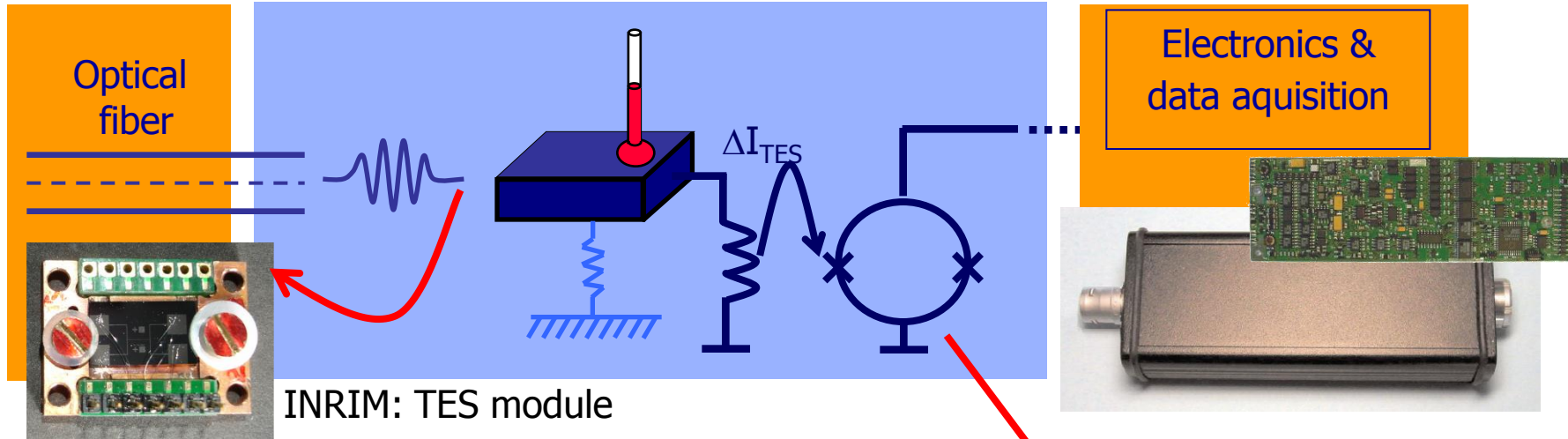
$$L_\beta = \sum_{\nu=1}^K f_\nu \log \left(\frac{p_0(\eta_\nu)}{\sum_{\lambda} p_0(\eta_\lambda)} \right) - \sum_{n=1}^N \beta \underbrace{n \langle n | \rho | n \rangle}_{\text{Diagonal elements of the density matrix}}$$

Fraction of 'no-click' events

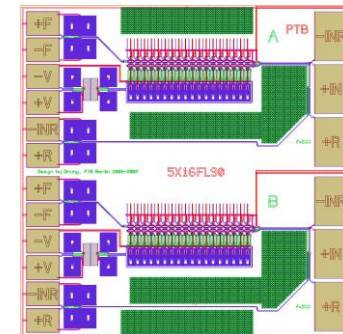
Diagonal elements of the density matrix



TES – Transition Edge Detector



20 μm x 20 μm



Nano-diamonds with NV-centres

